# Recognizing effective implementation of Standards for Mathematical Practices: Task choice, classroom practices, and teacher questions

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# Overview

- CCSS & NCTM
- Cognitive demand of tasks
- Practices associated with maintaining task difficulty
- Types of teacher questions
- Teacher questions as a tool to assess task difficulty, mathematical practices
- Preparing for class/observation with these tools

# Isn't there something awfully familiar?

- Haven't we heard these ideas before?

# CCSS Mathematical Practices, NCTM Process Standards

**Problem Solving**

1. Make sense of problems and persevere in solving them

5. Use appropriate tools strategically.

**Reasoning and Proof**

2. Reason abstractly and quantitatively.

3. Critique the reasoning of others.

8. Look for and express regularity in repeated reasoning.

NCTM *Summing It Up*, November 2010

# CCSS Mathematical Practices, NCTM Process Standards

**Communication**

3. Construct viable arguments.

**Connections**

6. Attend to precision.
7. Look for and make use of structure

**Representations**

4. Model with mathematics.

NCTM *Summing It Up*, November 2010

# Okay, so we HAVE seen this before

- Why is it important?

## Cognitive demands of tasks

- Low-Level Tasks
  - Memorization
  - Procedures without connections to meaning

Lack of Mathematical Practices

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Wealth of Mathematical Practices

- High-Level Tasks
  - Procedures with connections to meaning
  - Doing mathematics

Stein, Smith, Henningsen, & Silver (2000)

# Low-level task: Memorization

- Involve either producing previously learned facts, rules, formulae, or definitions *or* committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned to reproduced.

Stein, Smith, Henningsen, & Silver (2000)

# Low-level task: Procedures without connections

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedures being used.
- Are focused on producing correct answers rather than on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

Stein, Smith, Henningsen, & Silver (2000)

# High-level task: Procedures with connections

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are bread general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations help to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Stein, Smith, Henningsen, & Silver (2000)

# High-level task: Doing Mathematics

- Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Require students to explore and to understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge in working through the task.
- Require students to analyze the task and actively examining task constraints that may limit possible solutions strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Stein, Smith, Henningsen, & Silver (2000)

# Cognitive demands of tasks

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Wealth of Mathematical Practices

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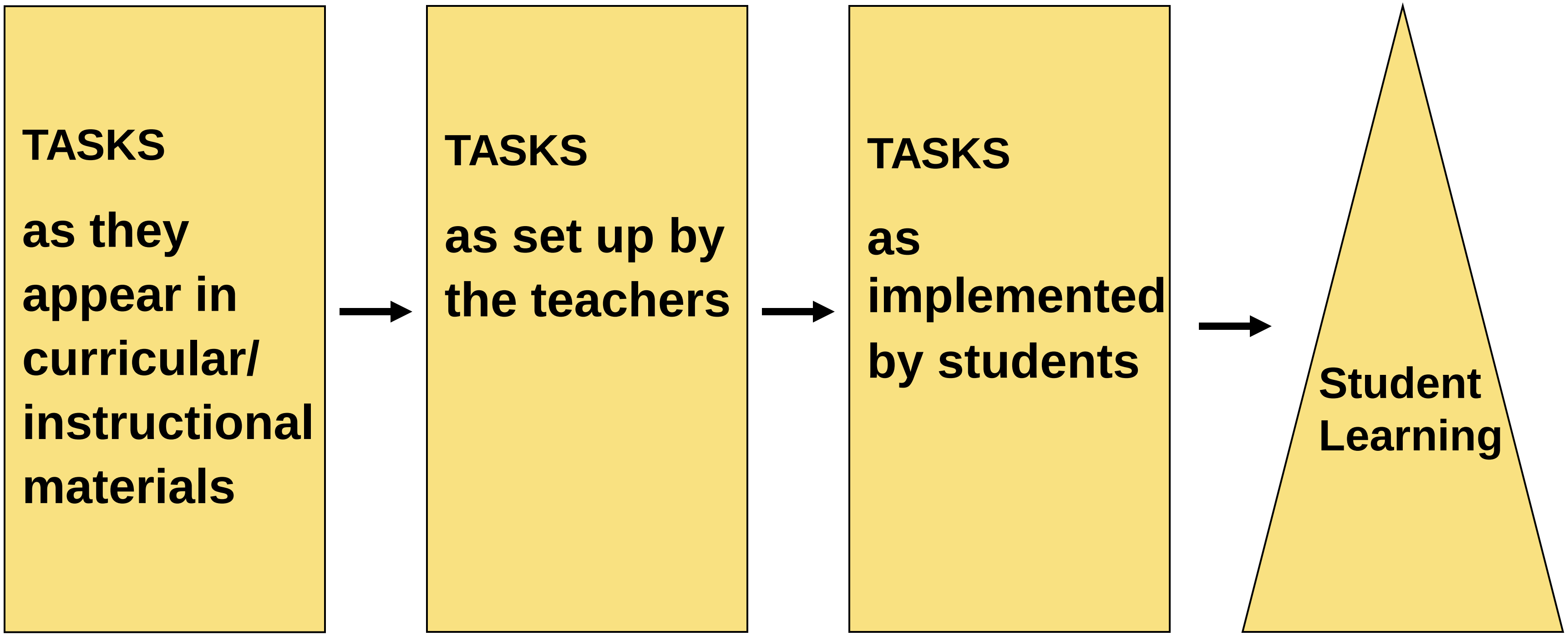
Stein, Smith, Henningsen, & Silver (2000)

# But we're not here to talk about tasks

- We're here to talk about teacher practices in the classroom.

- Why isn't choosing the right task enough?

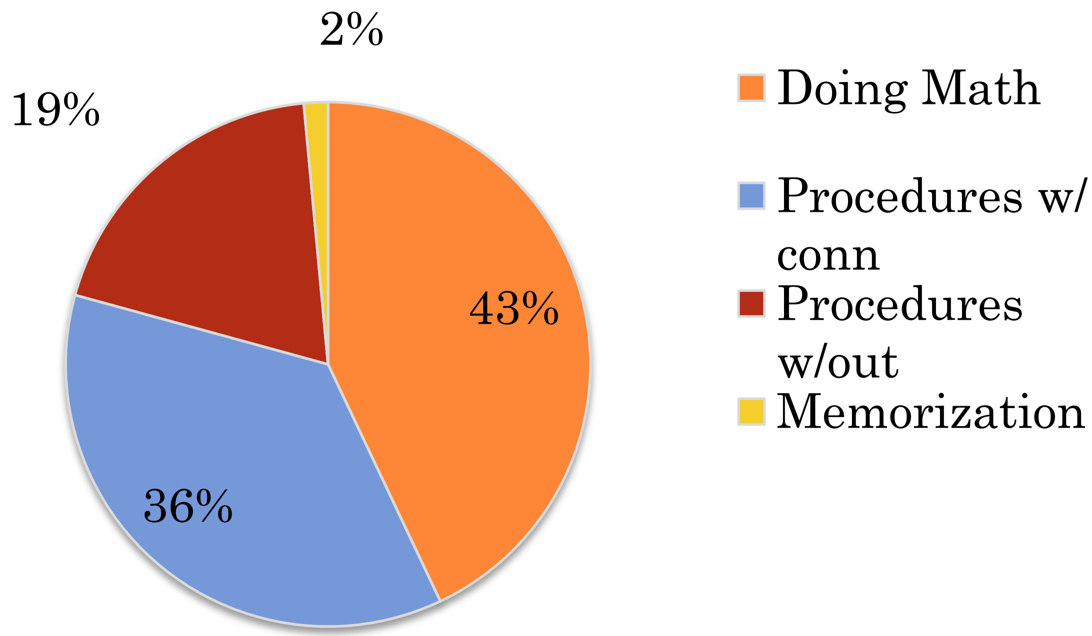
# Mathematical Tasks Framework




Stein, Smith, Henningsen, & Silver (2000)

# Stein, Grover, & Henningsen (1996)

- 144 Lessons, analyzed as set-up



# What happened in the classroom?

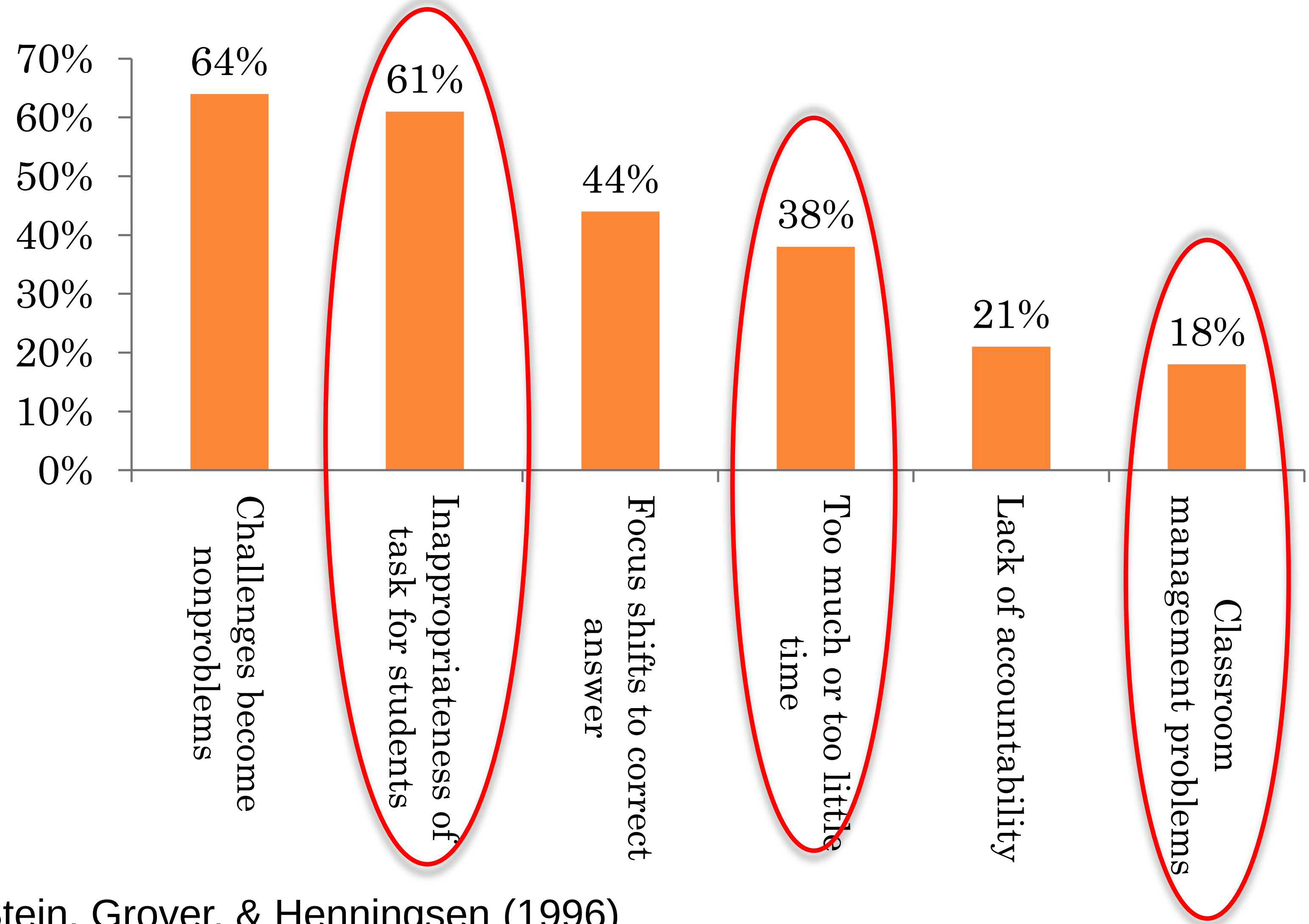
| | Implementation | | | | | | |
|---|---|---|---|---|---|---|---|
| Set up | No mathematical activity | Memorization | Procedures without connections | Procedures with connections | "Doing mathematics" | Other | Cannot discern |
| No mathematical activity required ($n$ = 3) | ***100%*** (3) | | | | | | |
| Memorization ($n$ = 2) | | ***50%*** (1) | | | 50% (1) | | |
| Procedures without connections ($n$ = 26) | | | ***96%*** (25) | 4% (1) | | | |
| Procedures with connections ($n$ = 49) | 2% (1) | 2% (1) | 53% (26) | ***43%*** (21) | | | |
| "Doing mathematics" ($n$ = 58) | 17% (10) | | 14% (8) | 3% (2) | ***38%*** (22) | 26% (15) | 2% (1) |
| Other ($n$ = 5) | | | | | | ***100%*** (5) | |
| Cannot discern ($n$ = 1) | | | 100% (1) | | | | |

Stein, Grover, & Henningsen (1996)

# So, the task is not enough.

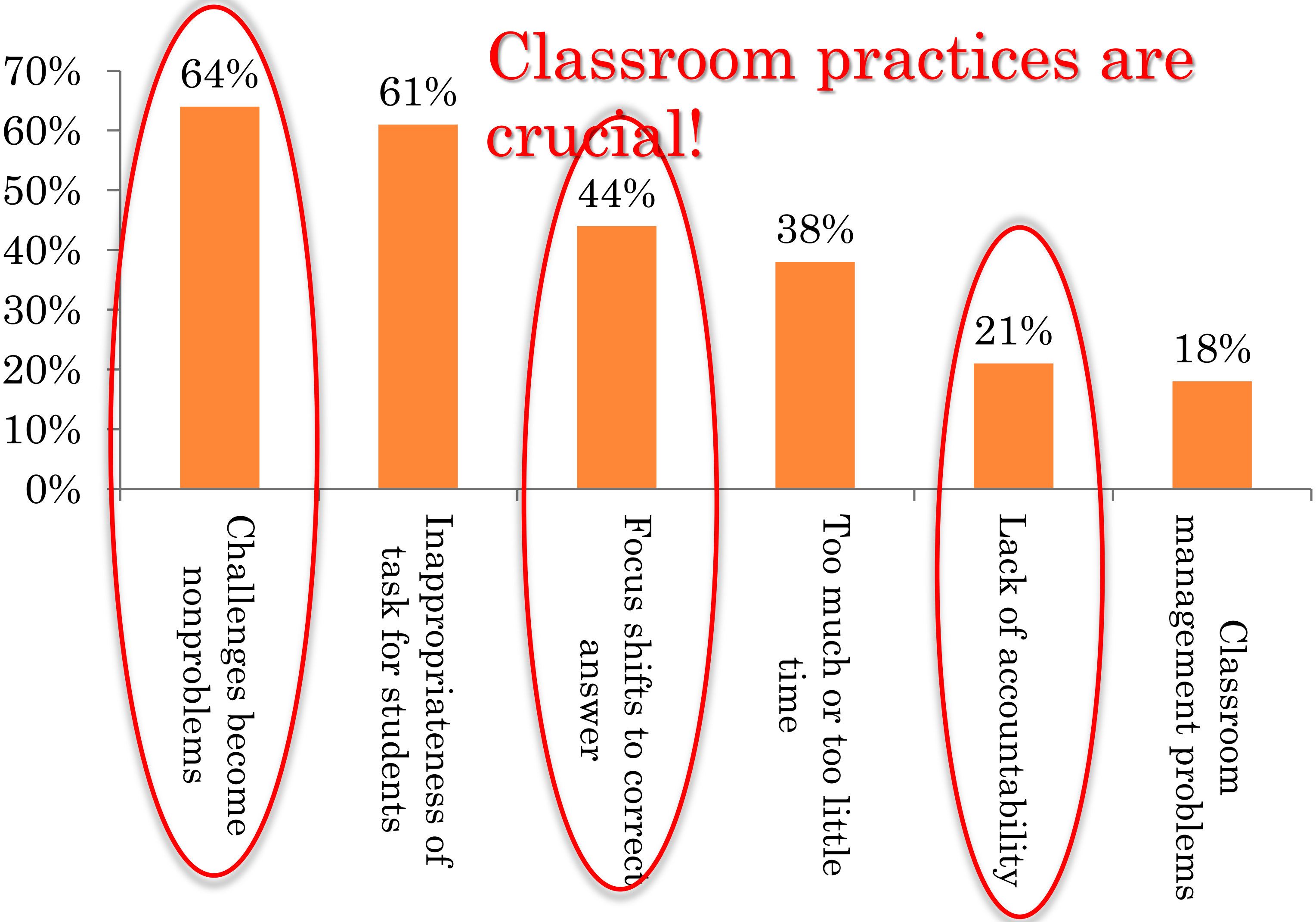
- What practices were associated with decreasing the difficulty of the tasks?

# PRACTICES THAT DECREASE DIFFICULTY



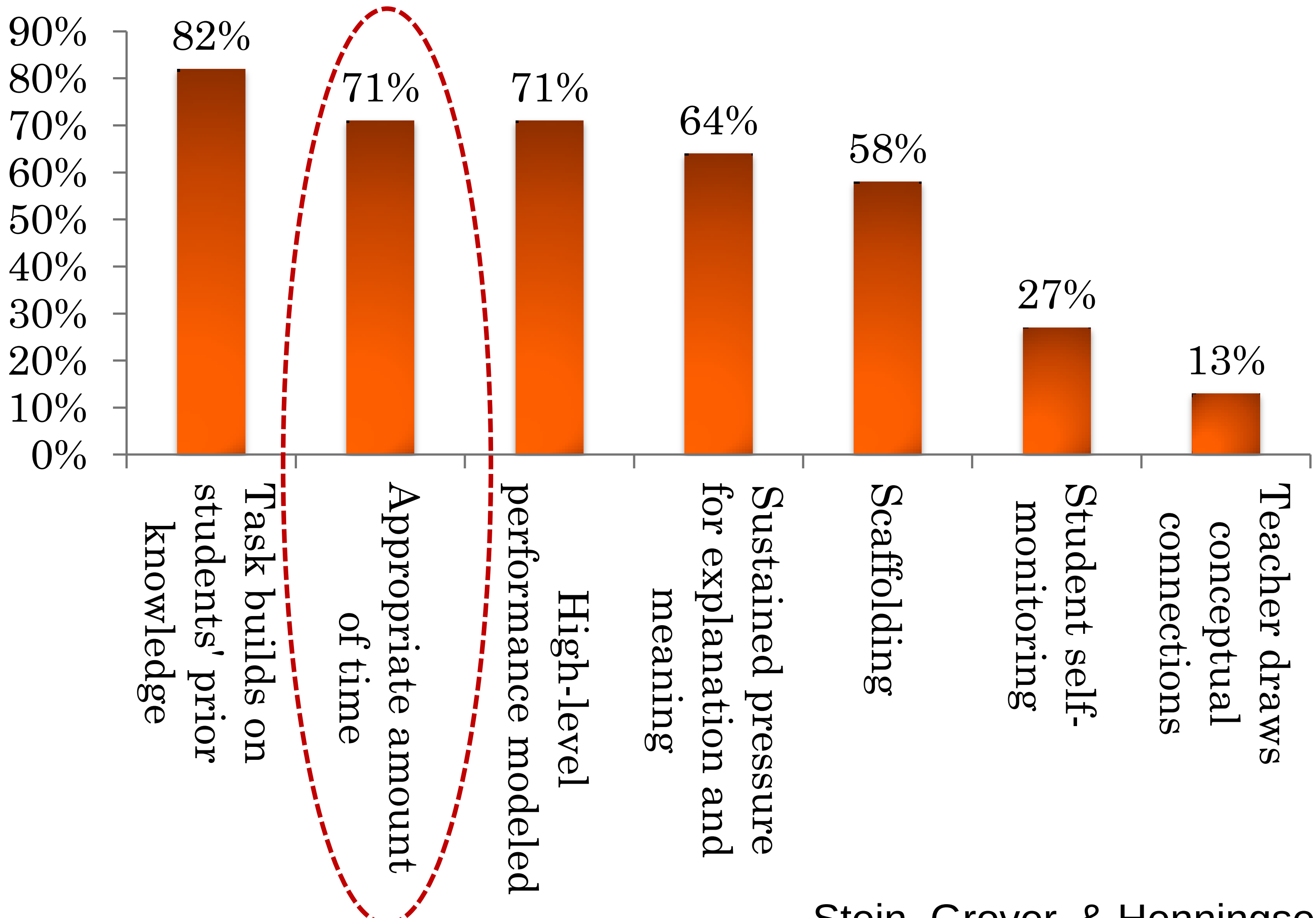

Stein, Grover, & Henningsen (1996)

# Practices that Decrease difficulty



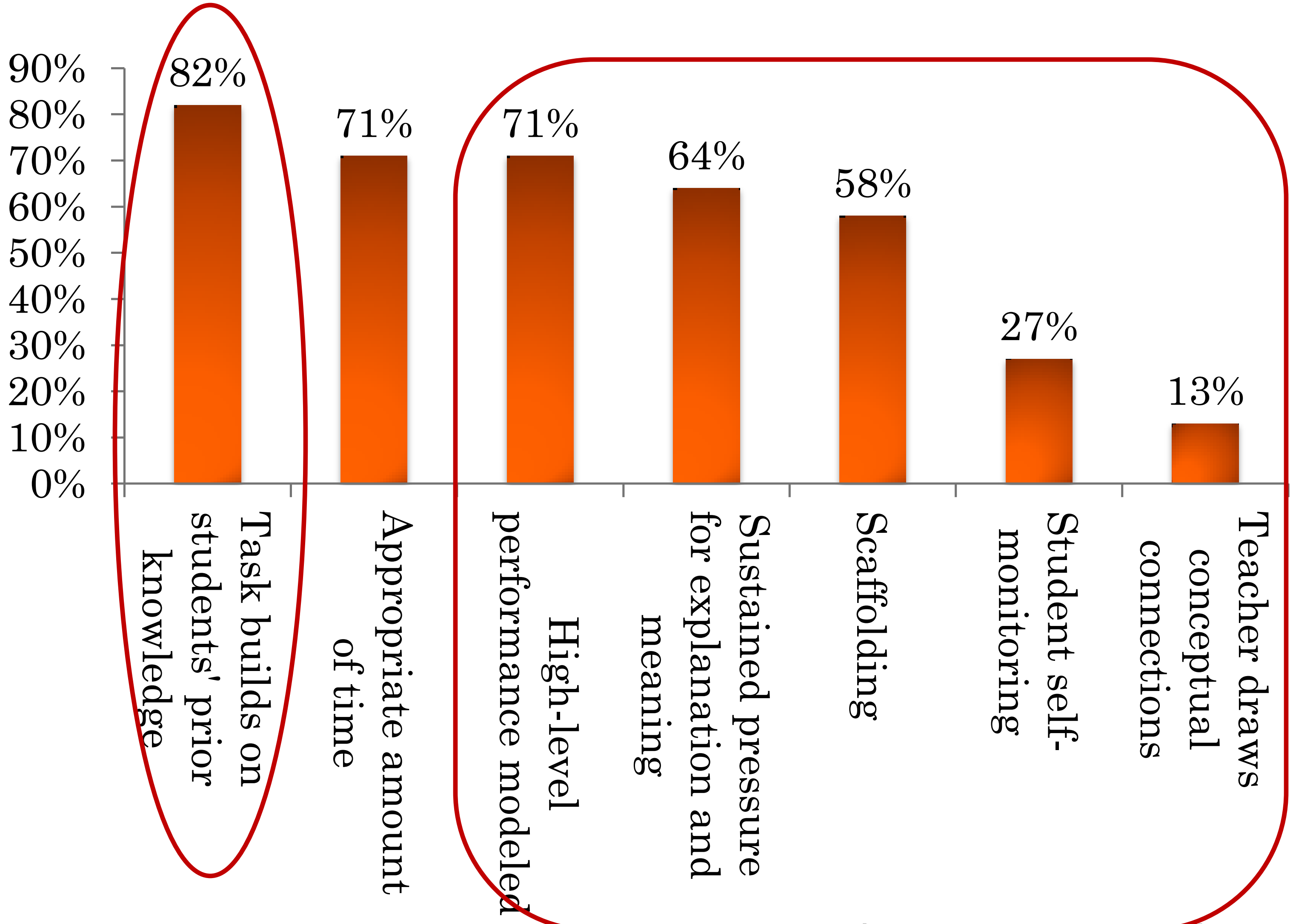

Stein, Grover, & Henningsen (1996)

# Practices that were associated with maintaining cognitive difficulty




Stein, Grover, & Henningsen (1996)

# PRACTICES THAT WERE ASSOCIATED WITH MAINTAINING COGNITIVE DIFFICULTY




Stein, Grover, & Henningsen (1996)

# What would we like to see in the classroom?

- Task builds on students' prior knowledge
- High-level performance modeled
- Sustained pressure for explanation and meaning
- Scaffolding
- Student self-monitoring
- Teacher draws conceptual connections

# Questioning – A tool for teachers, a guide for observers

- In order to observe these practices associated with maintaining task difficulty, what do we look for?

- Teacher questions can serve as hallmark for these practices.

# Types of Teacher Questions

1. Gathering information, leading students through a method
2. Inserting terminology
3. Exploring mathematical meaning and/or relationships
4. Probing, getting students to explain their thinking
5. Generating Discussion
6. Linking and applying
7. Extending thinking
8. Orienting and focusing
9. Establishing context

Boaler & Brodie (2004)

# 1. Gathering information, leading students through a method

Description:

- Requires immediate answer
- Rehearses known facts/procedures
- Enables students to state facts/procedures

E.g.

- What is the value of x in this equation?
- How would you plot that point?

Accounted for **97-99.5%** of teacher questions in classes with traditional tasks.

Accounted for only **61-71%** of teacher questions in classes with NCTM reform tasks.

Boaler & Brodie (2004)

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# 2. Inserting terminology

Description:

- Once ideas are under discussion, enables correct mathematical language to be used to talk about them

E.g.

- What is this called? How would we write this correctly?

Boaler & Brodie (2004)

# 3. Exploring mathematical meaning and/or relationships

Description:

- Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations

E.g.

- Where is this x on the diagram?
- What does probability mean?

Boaler & Brodie (2004)

# 4. Probing, getting students to explain their thinking

Description:

- Asks students to articulate, elaborate or clarify ideas

E.g.

- How did you get 10?
- Can you explain your idea?

Boaler & Brodie (2004)

# 5. Generating Discussion

Description:

- Solicits contributions from other members of class

E.g.

- Is there another opinion about this?
- What did you say, Justin?

Boaler & Brodie (2004)

# 6. Linking and applying

Description:

- Points to relationships among mathematical ideas and mathematics and other areas of study/life

E.g.

- In what other situations could you apply this?
- Where else have we used this?

Boaler & Brodie (2004)

# 7. Extending thinking

Description:

- Extends the situation under discussion to other situations where similar ideas may be used.

E.g.

- Would this work with other numbers?

Boaler & Brodie (2004)

# 8. Orienting and focusing

Description:

- Help students to focus on key elements or aspects of the situation in order to enable problem-solving

E.g.

- What is the problem asking you?
- What is important about this?

Boaler & Brodie (2004)

# 9. Establishing context

Description:

- Talks about issues outside of math in order to enable links to be made with mathematics

E.g.

- What is the lottery?
- How old do you have to be to play the lottery?

Boaler & Brodie (2004)

# Types of Teacher Questions

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Boaler & Brodie (2004)

# Identifying questions

| | |
|---|---|
| T: OK, not it says in there one student in solving this problem wrote C+ (C+20) + (C+40) = 90.Sothey didn't have an answer but they started writing some equation. So like the method over here was like guess and check. They were guessing some numbers and then they were checking them against the constraint, but somebody else decided to do their work C + (C+20) + (C+40) is 90. **What in the world is C?** Well, it has to do with this problem, obviously.<br><br>S: It's 10.<br><br>T: We've got C + (C+20) + (C+40) = 90. **What are you getting with 10? What's 10?** Who is it that said 10? | |

# Identifying questions

| | |
|---|---|
| S: I said 10. | |
| T: Jenny said 10. **Could you tell me how you got 10?** | 4 |
| S: A, 40 plus… plus… and then… yeah, yeah, 'cos 40 plus 10 is 50 and then 20 plus 10 is 30 and then 15 and 30 and then 10 more is 90. | |
| T: **So, you, did 10 just come out of a guess?** You were trying some numbers? | 4 |
| S: I got it wrong and then… | |

# Identifying questions

| | |
|---|---|
| T: But now you got it right? **Because on your homework, what'd you do?** | 4 |
| S: I said C was something else, I said C was 20. | |
| T: OK. So now she's saying 10 works. **Does everybody see why 10 works?** | 3 |
| T: OK So she said 10 plus 40 is 50, so we've got 50 here and 10 plus 20 she said was 30, and we've got 80 and we've got another 10 over here. **Now I'm curious why you put in 10 each time for C?** | 3 |
| S: Because C has to be the same. | |

# Identifying questions

| | |
|---|---|
| T: Okay. That's a really important point. C has to be the same each time so we couldn't have change it and made it a different number. OK, what is this topic called here? Up here. **What math topic is that called?**<br><br>S: Algebra. | 8 |
| T: That's called algebra. OK. So we're doing some algebra, in fact, this unit is out algebra unit. We're gradually getting there. **But what does C in words represent?** What in the world does this algebra equation have to do with this problem we're doing?<br><br>S: Ummm | 3 |

# Identifying questions

| | |
|---|---|
| T: What, what, what? **I mean how in the world did they get a bunch of these things and then they get this C junk with 30's and 40's?** | 8 |
| S: Urrrr. | |
| T: **Well, what could that possibly have to do with the problem?** | 8 |
| S: Uh? | |
| T: Jenny says the answer's 10. **Ten what?** Ten, ten's the number that. | 3 |

# Identifying questions

| | |
|---|---|
| S: Ten years? People? | |
| T: People? 10 years? **How does 10 years fit in with this problem?** Ten years we were on the trail? | 3 |
| S: Ten years old, our ages. Noo, the, children. | |
| T: Oh, children. | |
| S: It's the variable. | |
| T: **OK, so what's the variable?** | 4 |

# Connect to classroom practices associated with high-level task implementation

- Task builds on students' prior knowledge
- High-level performance modeled
- Sustained pressure for explanation and meaning
- Scaffolding
- Student self-monitoring
- Teacher draws conceptual connections

# 6, 7, and 8 Tend to run together

- For the next activity, these are lumped into "Linking, applying, extending, orienting, and focusing"

# Connect to Standards for Mathematical Practices

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

# How can I use this in my evaluation/teaching?

- Some types of questions do not vary based on lesson content
- Others depend on student input

- Keep these questions in mind, use a checklist sheet, note some of them that you hear

# Some questions we can expect to hear

- "Exploring mathematical meaning and/or relationships" is central to the mathematical goals of a lesson

- For each lesson, what mathematical meanings or relationships are central?
- What questions would you expect to hear from the teacher to highlight these ideas in class discussion?

## In review

- Task choice is related to the mathematical practices used.
- Task choice matters, but it is not enough to choose a good task.
- Teacher practices maintain or decrease the cognitive demand of tasks.
- We can use teacher questioning to evaluate cognitive demand and mathematical practices.

# Questions? Comments?

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