The Common Core State Standards for Mathematics: Hopes, Fears and Challenges as We Enter the Brave New World

MassMATE
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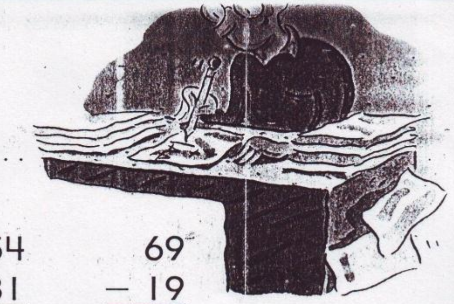
So…the problem is:

If we continue to do what we've always done….

We'll continue to get what we've always gotten.

Practice Plus



Key Skill: Subtraction, page 225

Subtract.

1.	32 − 17	48 − 23	86 − 67	54 − 31	69 − 19	
2.	77 − 46	51 − 27	98 − 25	66 − 33	40 − 16	83 − 55
3.	53 − 26	31 − 8	74 − 26	95 − 48	57 − 19	21 − 10

Key Skill: Subtracting Money, page 23

Subtract.

1.	27¢ − 18¢	31¢ − 26¢	44¢ − 27¢	63¢ − 29¢	97¢ − 59¢	80¢ − 41¢
2.	49¢ − 22¢	74¢ − 59¢	89¢ − 39¢	56¢ − 37¢	65¢ − 48¢	92¢ − 15¢
3.	38¢ − 29¢	52¢ − 13¢	84¢ − 68¢	67¢ − 43¢	75¢ − 27¢	99¢ − 11¢

One seven-year-old student's viewpoint of life at 100

If I were 100 years old

If I were 100 years old,
I would go to a nursing
home. I would stay
there until I was dead.
By the time I was 100,
I would know Regrouping
with sabtraction and
then I would die happy.

Alg II L3 Exam Do not write on this exam. 25

I Perform the indicated operation and simplify.

1. $\frac{9x^7}{36x^{10}y^3}$ a) $\frac{3}{12x^3y^3}$ b) $\frac{x^7}{4x^{10}y^3}$ c) $4x^3y^3$ d) $\frac{1}{4}x^3y^3$ e) $\frac{1}{4x^3y^3}$

2. $\frac{x^2-x-12}{x^2+10x+21}$ a) $\frac{x-4}{x+7}$ b) $\frac{x+4}{x+7}$ c) $\frac{1-x-4}{1+10+x}$ d) $\frac{4}{7}$ e) $\frac{(x+12)(x-1)}{(x+3)(x+7)}$

3. $\frac{3x^4}{4a^2y^2} \cdot \frac{8a^6y^3}{6x^6}$ a) $\frac{a^2y^3x^{-2}}{y^2}$ b) $\frac{2a^4y}{2x^2}$ c) $\frac{a^4y}{x^2}$ d) $24a^8y^5$ e) $\frac{9x^{10}}{16a^8y^5}$

4. $\frac{x^2-3x-10}{x^2-4} \cdot \frac{x-2}{x-5}$ a) 0 b) $\frac{x+2}{x-2}$ c) 1 d) $\frac{x-5}{x-2}$ e) $\frac{x^3-3x^2+20}{x^3+20}$

5. $\frac{36x^2y^2}{15b^3z} \div \frac{[illegible]y^4}{5[illegible]z^3}$ a) $\frac{2z^2}{3b^2xy^2}$ b) $\frac{3b^2z^2}{6xy^2}$ c) $\frac{2b^2z^2}{xy^2}$ d) $\frac{2z^2}{b^2xy^2}$ e) $\frac{2}{[illegible]^2b^2}$

6. $\frac{x^2+4x+3}{x^2-y^2} \div \frac{x+3}{x-y}$ a) 0 b) $\frac{x+1}{x+y}$ c) $\frac{x+1}{x-y}$ d) $\frac{x-1}{x+y}$ e) $\frac{1}{y}$

7. $\frac{8x^2-3x+2}{x^3} + \frac{x^2+5x-4}{x^3}$ a) $\frac{9x^2+2x+2}{x^3}$ b) $\frac{8x^4+2x-2}{x^6}$ c) $\frac{9x^2+2x-2}{2x^3}$
d) $\frac{9x^2+2x-2}{x^3}$ e) $\frac{8x^2-8x-6}{x^3}$

8. $\frac{4y+3}{y-2} - \frac{y-2}{y-2}$ a) $\frac{3y+1}{y-2}$ b) $\frac{3y+5}{y-2}$ c) 0 d) $\frac{5y+5}{y-2}$ e) $\frac{5y+1}{y-2}$

9. $\frac{5x}{3} + \frac{4y}{5}$ a) 0 b) $25x+12y$ c) $\frac{37xy}{15}$ d) $\frac{5x+4y}{8}$ e) $\frac{25x+12y}{15}$

10. $\frac{4}{x} - \frac{5}{y}$ a) -1 b) $\frac{4x-5y}{xy}$ c) $\frac{5x+4y}{xy}$ d) $-\frac{1}{xy}$ e) $\frac{4y-5x}{xy}$

11. $\frac{18x^5+24x^4-36x^3}{6x^2}$ a) $3x^3+4x^2-6x$ b) $12x^3+18x^2-30x$ c) $3x^3+4x^2+6x$ d) $12x^3+18x-30$ e) $54x^7+144x^6-216x^5$

12. $x-2\overline{\smash{)}x^2-12x+20}$ a) $x-10$ b) $x+10$ c) $x+10\ \frac{2}{x-2}$ d) $x-10\ \frac{-2}{x-2}$ e) $x-42$

II Solve for x

13. $\frac{x}{3} + \frac{x}{4} = 21$ a) 36 b) 3 c) 126 d) $\frac{3}{4}$ e) $83\frac{1}{3}$

14. $\frac{3}{x+1} = \frac{2}{x-3}$ a) b) 1 c) -4 d) $\pm 3\sqrt{2}$ e) 9

15. $\frac{3}{4x} + \frac{1}{10} = \frac{3}{2x}$ a) $-\frac{2}{3}$ b) $\frac{3}{2}$ c) $-\frac{3}{2}$ d) $\frac{3}{3}$ e) 0

III Simplify each of the following

16. 8^0 a) 1 b) 0 c) 5 d) 8 e) not possible

17. $5x^0$ a) 0 b) 5 c) 1 d) $5x$ e) 50

IV Write without negative exponents and simplify

18. $\frac{3xy^{-3}}{9a^{-2}b^2}$ a) $\frac{3x y^{-3}}{9b^2 a^{-2}}$ b) $\frac{3a^2x}{9b^2y^3}$ c) $\frac{a^2x}{3b^2y^3}$ d) $\frac{\frac{3x}{y^3}}{\frac{9b^2}{a^2}}$ e) None of These

10. $\dfrac{4}{x} - \dfrac{5}{y}$ a) -1 b) $\dfrac{4x-5y}{xy}$ c) $\dfrac{5x+4y}{xy}$ d) $-\dfrac{1}{xy}$ e) $\dfrac{4y-5x}{xy}$

11. $\dfrac{18x^5+24x^4-36x^3}{6x^2}$ a) $3x^3+4x^2-6x$ b) $12x^3+18x^2-30x$ c) $3x^3+4x^2+6x$ d) $12x^2\ 18x-30$ e) $54x^7+144x^6-216x^5$

12. $x-5 \overline{) x^2-15x+50}$ a) $x-10$ b) $x+10$ c) $x+10\ \dfrac{5}{x-5}$ d) $x-10\ \dfrac{-5}{x-5}$ e) $x-45$

II Solve for x

13. $\dfrac{x}{3}+\dfrac{x}{4}=21$ a) 36 b) 3 c) 126 d) $\dfrac{3}{2}$ e) $82\ ^2/_3$

14. $\dfrac{3}{x+1}=\dfrac{2}{x-3}$ a) b) 1 c) -4 d) $\pm 3\sqrt{2}$ e) $\emptyset$

15. $\dfrac{3}{4x}+\dfrac{1}{10}=\dfrac{3}{5x}$ a) $-\dfrac{2}{3}$ b) $\dfrac{3}{2}$ c) $-\dfrac{3}{2}$ d) $\dfrac{2}{3}$ e) 0

III Simplify each of the following

16. 8^0 a) 1 b) 0 c) 2 d) 8 e) not possible

17. $2x^0$ a) 0 b) 2 c) 1 d) $2x$ e) 20.

IV Write without negative exponents and simplify

18. $\dfrac{3xy^{-3}}{2a^{-2}b^3}$ a) $\dfrac{3x\cdot y^3}{2b^3a^2}$ b) $\dfrac{3a^2x}{2b^2y^3}$ c) $\dfrac{6a^2x}{3b^3y}$ d) $\dfrac{\dfrac{3x}{y^3}}{\dfrac{2b^2}{a^2}}$ e) None of These

Algebra:

The intense study of the last three letters of the alphabet

So what have we gotten?

- **Mountains of math anxiety**
- **Tons of mathematical illiteracy**
- **Mediocre test scores**
- **HS programs that barely work for more than half of the kids**
- **Gobs of remediation and intervention**
- **A slew of criticism**

Not a pretty picture!

If however…..

What we've always done is no longer acceptable, then…

We have no choice but to change some of what we do and some of how we do it.

So what does different mean?

Some data. What do you see?

	40	4
	10	2
	30	4

Predict some additional data

	40	4
	10	2
	30	4

How close were you?

	40	4
	10	2
	30	4
	20	3

All the numbers – so?

	45	4
	25	3
	15	2
	40	4
	10	2
	30	4
	20	3

A lot more information (where are you?)

Roller Coaster	45	4
Ferris Wheel	25	3
Bumper Cars	15	2
Rocket Ride	40	4
Merry-go-Round	10	2
Water Slide	30	4
Fun House	20	3

Fill in the blanks

Ride	???	???
Roller Coaster	45	4
Ferris Wheel	25	3
Bumper Cars	15	2
Rocket Ride	40	4
Merry-go-Round	10	2
Water Slide	30	4
Fun House	20	3

At this point,

it’s almost anticlimactic!

The amusement park

Ride	Time	Tickets
Roller Coaster	45	4
Ferris Wheel	25	3
Bumper Cars	15	2
Rocket Ride	40	4
Merry-go-Round	10	2
Water Slide	30	4
Fun House	20	3

The Amusement Park

The 4^{th} and 2^{nd} graders in your school are going on a trip to the Amusement Park. Each 4^{th} grader is going to be a buddy to a 2^{nd} grader.

Your buddy for the trip has never been to an amusement park before. Your buddy want to go on as many different rides as possible. However, there may not be enough time to go on every ride and you may not have enough tickets to go on every ride.

The bus will drop you off at 10:00 a.m. and pick you up at 1:00 p.m. Each student will get 20 tickets for rides.

Use the information in the chart to write a letter to your buddy and create a plan for a fun day at the amusement park for you and your buddy.

Why do you think I started with this task?

- Standards don't teach, teachers teach
- It's the translation of the words into tasks and instruction and assessments that really matter
- Processes are as important as content
- We need to give kids (and ourselves) a reason to care
- Difficult, unlikely, to do alone!!!

Today's Goal

To provoke and inform your thinking about the need to shift our curriculum, instructional practices, professional culture, and mindsets in ways that are aligned with the vision of the new Common Core State Standards and that truly meet the needs of all students.

Today's Agenda

- **Five perspectives on our reality**
- **Some glimpses of the CCSSM**
- **Some glimpses of a CCSSM future**
- **Some fears and challenges**

(That is, a bombardment in 4 parts)

1. What a great time to be convening as teachers of mathematics!

- Common Core State Standards adopted by 45 states
- Quality K-8 instructional materials
- More access to material and ideas via the web than ever
- A president who believes in science and data
- The beginning of the end to Algebra II as the killer
- A long overdue understanding that it's instruction that really matters
- A recognition that the U.S. doesn't have all the answers

2. Where we live on the food chain

Economic security and social well-being

↑ ↑ ↑

Innovation and productivity

↑ ↑ ↑

Human capital and equity of opportunity

↑ ↑ ↑

High quality education

(literacy, MATH, science)

↑ ↑ ↑

Daily classroom math instruction

3. Let’s be clear:

We’re being asked to do what has never been done before:

Make math work for nearly ALL kids and get nearly ALL kids ready for college.

There is no existence proof, no road map, and it’s not widely believed to be possible.

4. Let’s be even clearer:

Ergo, because there is no other way to serve a much broader proportion of students:

We’re therefore being asked to teach in distinctly different ways.

Again, there is no existence proof, we don’t agree on what “different” mean, nor how we bring it to scale.

(That’s the hope of the CCSSM for Math)

5. So the key things we know

People won’t do what they can’t <u>envision</u>,

People can’t do what they don’t <u>understand</u>,

People can’t do well what isn’t <u>practiced</u>,

But practice without <u>feedback</u> results in little change, and

Work without <u>collaboration</u> is not sustaining.

Ergo: Our job, as professionals, at its core, is to help people envision, understand, practice, receive feedback and collaborate.

Non-negotiable take-away:

Make no mistake,

for K-12 math in the U.S., this

(the advent of Common Standards)

IS brave new world.

Full disclosure

For better or worse, I've been drinking the CCSSM Kool-aid.

Leinwand on the CCSSM in the 2011 Heinemann catalog.

A Long Overdue Shifting of the Foundation

For as long as most of us can remember, the K-12 mathematics program in the U.S. has been aptly characterized in many rather uncomplimentary ways: underperforming, incoherent, fragmented, poorly aligned, unteachable, unfair, narrow in focus, skill-based, and, of course, "a mile wide and an inch deep." Most teachers are well aware that there have been far too many objectives for each grade or course, few of them rigorous or conceptually oriented, and too many of them misplaced as we ram far too much computation down too many throats with far too little success. It's not a pretty picture and helps to explain why so many teachers and students have been set up to fail and why we've created the need for so much of the intervention that test results seem to require.

But hope and change have arrived! Like the long awaited cavalry, the new *Common Core State Standards (CCSS) for Mathematics* presents a once in a lifetime opportunity to rescue ourselves and our students from the myriad curriculum problems we've faced for years.

COHERENT FAIR TEACHABLE

Kool-aid (for you youngsters)

- **The flavored crystals you mix with water and put in the large pitcher because it's cheaper than soda.**
- **As in Tom Wolfe's "The Kool-aid Acid Test", the liquid used to mix the LSD among Ken Kesey and the Merry Pranksters.**
- **The liquid Jim Jones and his cult members used to ingest the mass suicidal poison.**

In all three cases, nothing to be proud of.

So finally, let's take a look at the game changer:

The Common Core State Standards for Mathematics

Promises

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

— CCSSM (2010, p.5)

Some design elements of the CCSS

- Fewer, clearer, higher
- Fairer – rational grade placement of procedures
- NCTM processes transformed into mathematical practices
- Learning trajectories or progressions
- Spirals of expanding radius – less repetitiveness and redundancy
- A sequence of content that results in all students reaching reasonable algebra in 8th grade
- Balance of skills and concepts – what to know and what to understand

CCSSM Grade 2 (pp. 17-20)

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Add and subtract within 20.**
- **Work with equal groups of objects to gain foundations for multiplication.**

Number and Operations in Base Ten

- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure and estimate lengths in standard units.**
- **Relate addition and subtraction to length.**
- **Work with time and money.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

GRADE 2: Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.

7. <u>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.</u> Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

9. <u>Explain why addition and subtraction strategies work, using place value and the properties of operations</u>

CCSSM Grade 6 (pp. 41-45)

Ratios and Proportional Relationships

- **Understand ratio concepts and use ratio reasoning to solve problems.**

The Number System

- **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
- **Compute fluently with multi-digit numbers and find common factors and multiples.**
- **Apply and extend previous understandings of numbers to the system of rational numbers.**

Expressions and Equations

- **Apply and extend previous understandings of arithmetic to algebraic expressions.**
- **Reason about and solve one-variable equations and inequalities.**
- **Represent and analyze quantitative relationships between dependent and independent variables.**

Geometry

- **Solve real-world and mathematical problems involving area, surface area, and volume.**

Statistics and Probability

- **Develop understanding of statistical variability.**
- **Summarize and describe distributions.**

Grade 6: Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. ***For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."***

2. Understand the concept of a unit rate ***a/b associated with a ratio a:b*** **with** ***b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."***

3. <u>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</u>

a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. ***For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?***

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

CCSSM Mathematical Practices

The Common Core proposes a set of Mathematical Practices that all teachers should develop in their students. These practices are similar to NCTM's Mathematical Processes from the *Principles and Standards for School Mathematics*.

8 CCSSM Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**

8 CCSSM Mathematical Practices

5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Some glimpses of the future (the real hope and implications)

Dream with me

- Assessment as a process for gathering evidence to support claims about students
- Computer adaptive assessments with routers and appropriate items for stage 2
- 4 week testing windows
- Computer-scored constructed response items
- Item banks for formative, benchmark AND summative assessments
- Student, class, school, district, state and nation results three days after the window closes

Hong Kong – Grade 3 - 2007

Fill in the boxes with "$>$", "$<$" or "$=$".

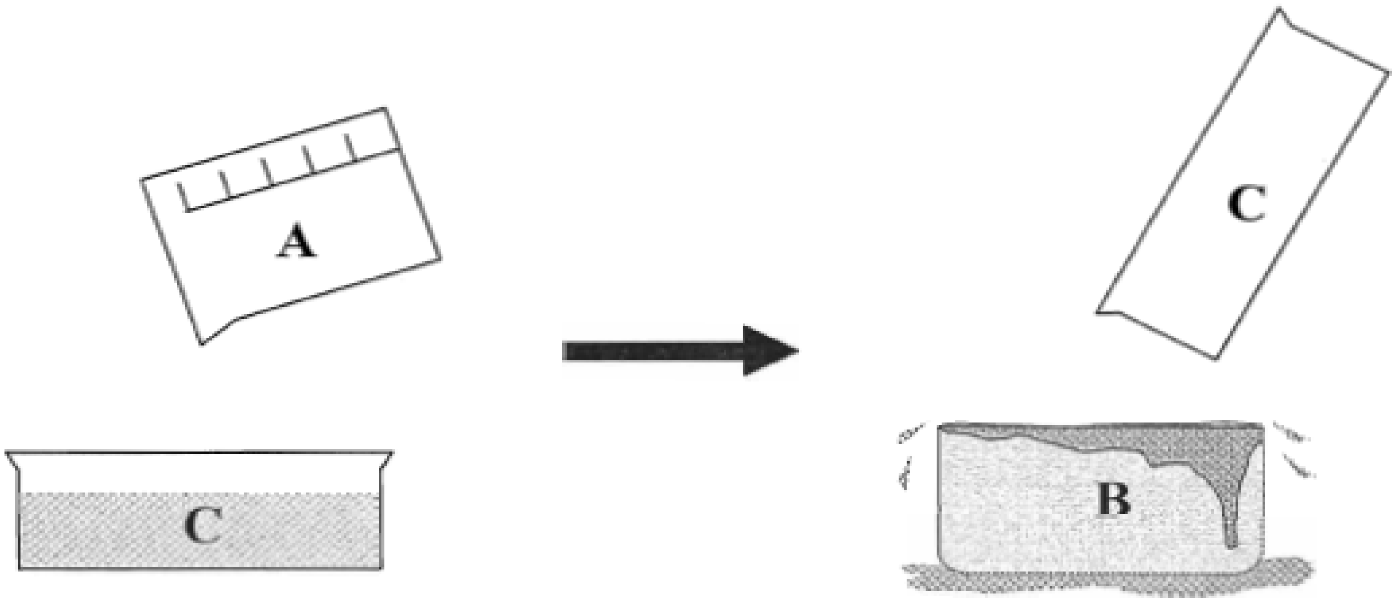
(a) $1 \ \square \ \frac{10}{10}$

(b) $\frac{1}{5} \ \square \ \frac{1}{8}$

(c) $\frac{3}{11} \ \square \ \frac{9}{11}$

(d) $\frac{2}{2} \ \square \ 2$

Hong Kong – Grade 3 - 2007



Container A is filled completely with water and all the water is poured into Container C. Then all the water in Container C is poured into Container B (the results are shown in the diagram above). Arrange Containers A, B and C in order, from the largest capacity to the smallest. Write the letters for the answer.

Answer: __________, __________, __________
(Largest) (Smallest)

China 9-12 Standards E.g. (http://hrd.apec.org)

Example 4 According to weather forecast, the probability that there will be a small flood in some place next month is 0.25, and that the probability for a big flood is 0.01. Suppose that there is a piece of large-scale equipment in a construction site and there are three plans to protect it from the floods.

Plan 1: Remove the equipment. This costs 3800 dollars.

Plan 2: Construct a wall for protection. This costs 2000 dollars. But this wall cannot stop the big flood. When the big flood comes and the equipments are destroyed, the loss is 60000 dollars.

Plan 3: No plan at all. Hope that no flood will come. When there is a big flood the loss is 60000 dollars. When there is a small flood the loss is 10000 dollars.

Try to compare and decide which plan is better.

Outstanding HS Issues

- Some vs. All
  - 113 for all + 43 (*) for STEM nerds
- Traditional vs. Integrated
  - “Pathways”
- Grade 8 Algebra
  - When can we skip? Or should we double up?
- Catching up
  - Boost-up interventions

Rollout Timeline, hopefully

- **2010-11: A year of comprehensive planning (clarifying what needs to be done when)**
- **2011-12: A year of study (analyzing crosswalks, curricular implications, policy shifts)**
- **2012-13: A year of piloting and collaborative discussions**
- **2013-14: A year of curriculum and policy implementation and an assessment moratorium**
- **2014-15: A year of accountable implementation**

Next steps

- Build familiarity
- Grade by grade and course by course discussions
- Professional collaboration
- Crosswalks
- The mathematical practices
- Think 10% per year
- Focus on instructional quality/opportunity to learn

What's to be afraid of?

- Ok – we've got the standards, go do 'em
- Another fad with political intrusion
- Doomed by the same lack of capacity that got us into this mess in the first place
- Assessment compromises
- Not enough time gel
- We forget that it's instruction stupid
- We ignore the essential roles of school and department culture

But what's to be so hopeful about?

- Systemic alignment of standards, materials, assessments and pd
- Less attention to WHAT math and more attention to HOW to best teach it
- Greater collaboration around clearer and common goals and better data
- Market incentives for technology and video
- A chance to finally focus primarily in instruction and learning

So….

While we acknowledge the range and depth of the problems we face,

It should be comforting to see the availability and potential of solutions to these problems….

Now go forth and start shifting YOUR school’s mathematics program to better serve our students, our society and our future.

Thank you!

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