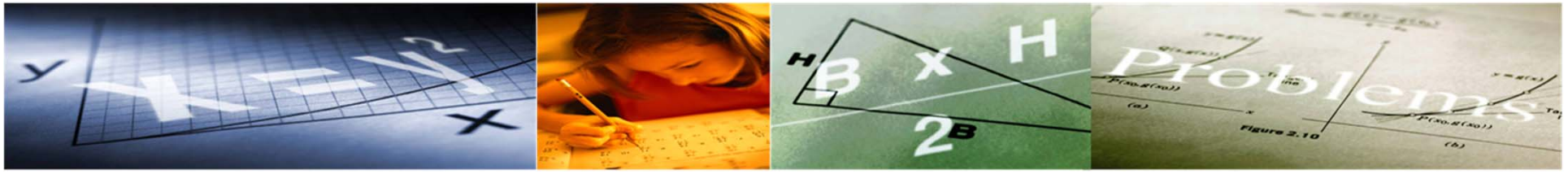
Helping Teachers Transition to the New Standards for Mathematical Practice PreK-5

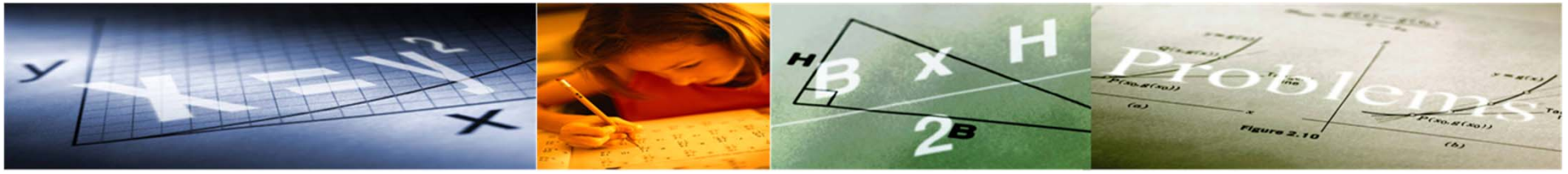
MassMATE Symposium

May 23, 2012



How can Teacher Leaders Help Colleagues Transition to the New Standards for Mathematical Practice?

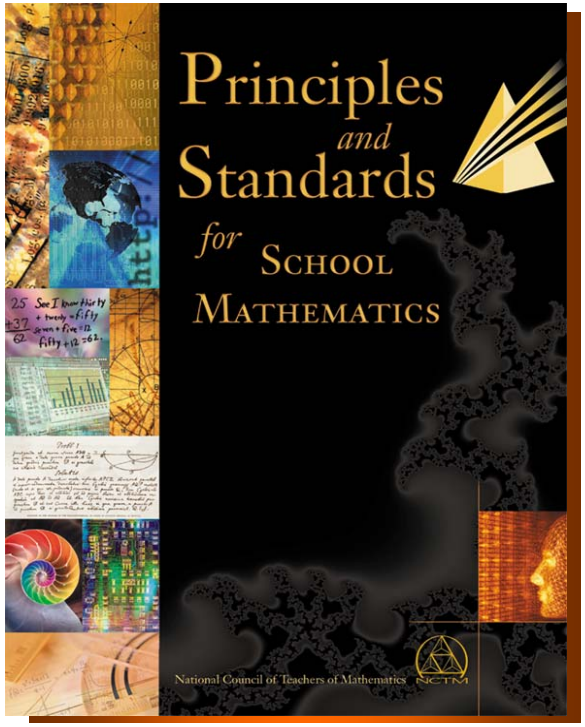
- Analysis of Student Work
- Content that Brings Clarity to Mathematical Practices
- Task Analysis



National Council of Teachers of Mathematics

5 **Process** Standards

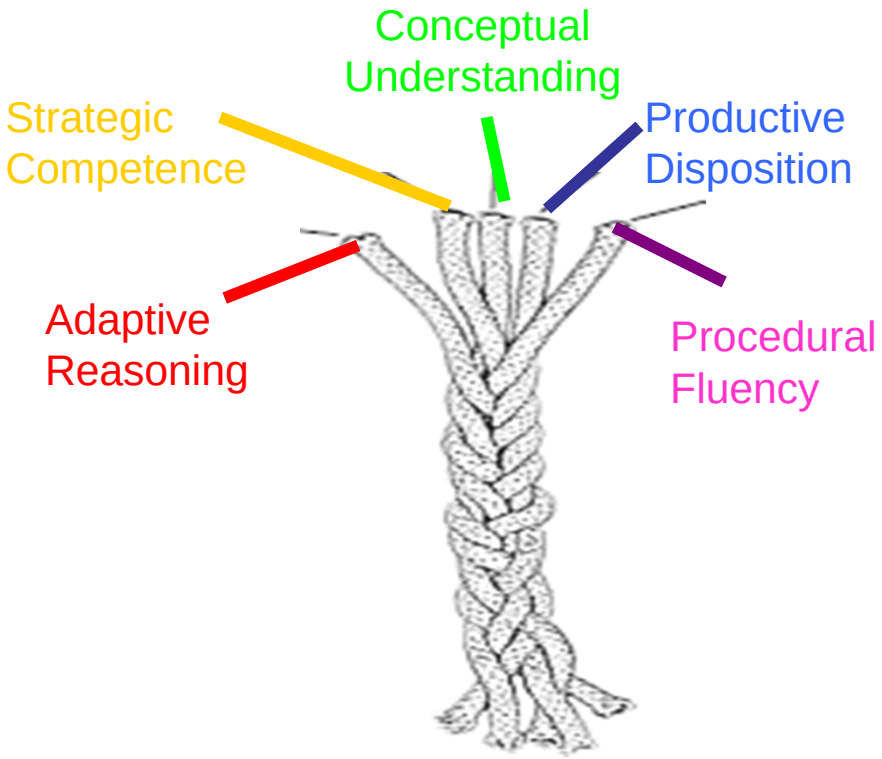
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

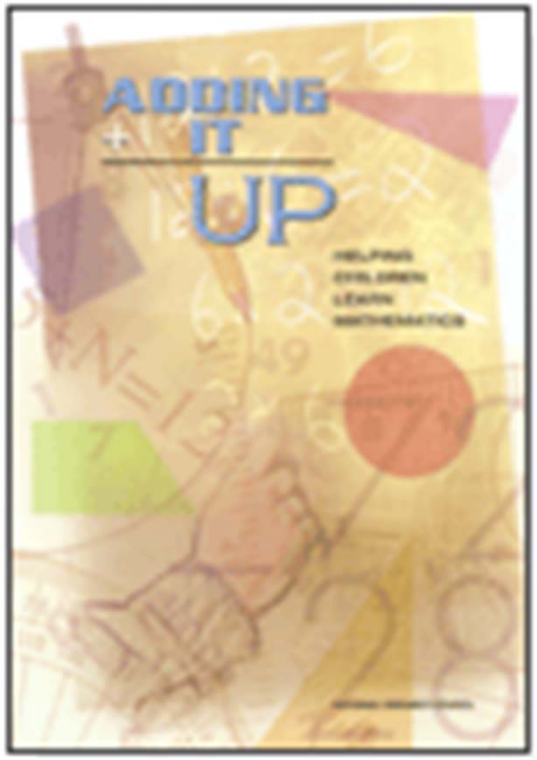


NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Source: NCSM/Standards for Mathematical Practice: Getting Started

Strands of Mathematical Proficiency





NRC (2001). *Adding It Up*. Washington, D.C.: National Academies Press.

Source: NCSM/Standards for Mathematical Practice: Getting Started

Verbs in the Standards for Mathematical Practice

Non measurable

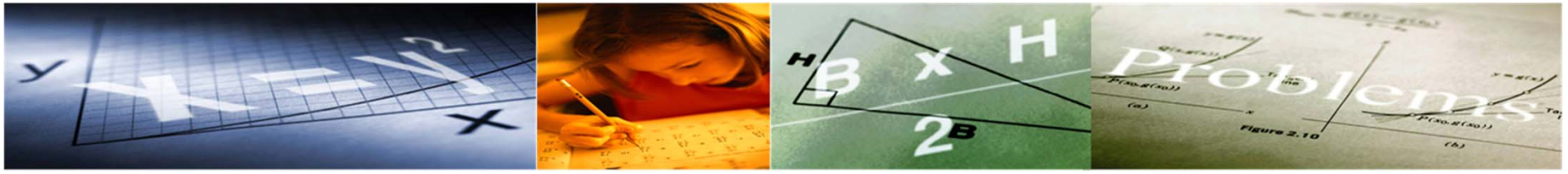
1. *Explain and make conjectures..*
2. *Make sense of…*
3. *Understand and use…*
4. *Apply and interpret…*
5. *Consider and detect…*
6. *Communicate precisely to others…….*
7. *Discern and recognize…*
8. *Notice and pay attention to…*

By their ability to……..

1. *Find the meaning of a problem and select an entry point*
2. *Create a coherent representation, make sense of quantities*
3. *Test assumptions, build on ideas*
4. *Write and solve equations and describe situations and actions in problems*
5. *Select available tools and consider reasonableness, estimate*
6. *Understand symbols and specify units*
7. *Generalize (7 + 3 = 3 + 7)*
8. *Use repeated reasoning (adding or subtracting zero)*

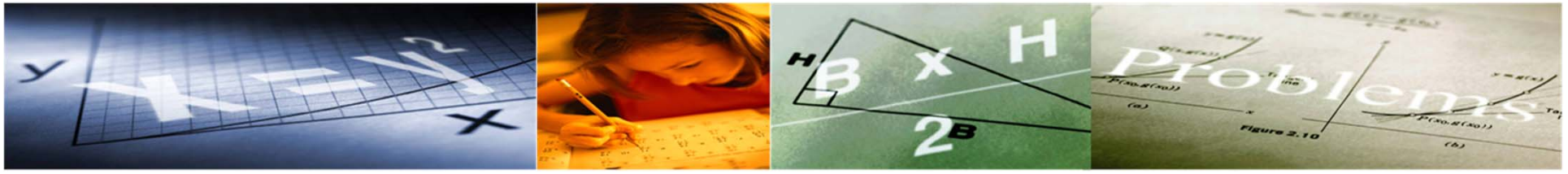
Understanding Mathematical Practices by Analysis of Student Work

- Jackie and Sara collected shells at the beach. Jackie collected 48 shells in all. She collected 13 more shells than Sara. How many shells did Sara collect? Use pictures, words, or numbers to show how you got your answer.



Evidence of Mathematical Practices

- MP 1. Making sense
- M P 2. Quantities and relationship to the task
- MP 4. Models and representations
- MP 6. Explaining with precision



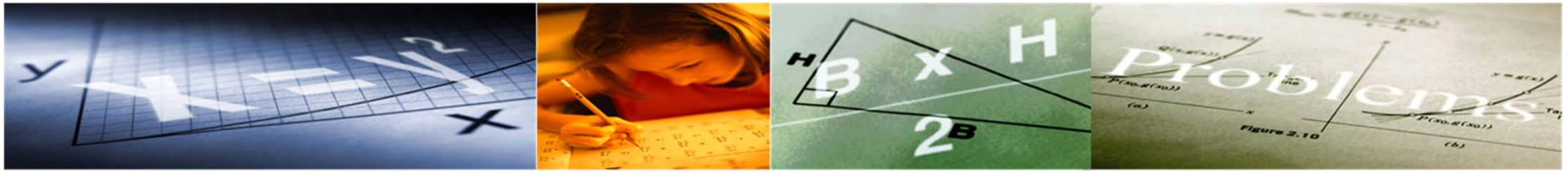
Analyze the Work ……..

- What strategy does the student use to solve the problem?
- What evidence exists that the student understands the connection between addition and subtraction?
- If the student drew a picture, does the student use it to solve the problem or represent the solution?
- What other Mathematical Practices are evident in the student work?

Bringing Clarity to Mathematical Practices via Content Standards

Properties of operations, relationship between addition and subtraction, unknowns in all positions……..

- PK.OA.1
- K.OA.1 – 5
- 1.OA.1 – 8 1.NBT.4 & 6
- 2.OA.1 & 2 2.NBT.5,7 & 9 2.MD.5
- 3.OA.8 3.NBT.2
- 4.NF.3c-d
- 5.NBT.7



Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[127]
Put Together/ Take Apart[128]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[129]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Bringing Clarity to Mathematical Practices via Task Analysis

A traditional problem involving operations with fractions would look something like this:

> Fill in the blank with <, >, or =.
>
> $\frac{3}{5}$ ____ $\frac{7}{10}$

MP6: When students are comparing two fractions with unlike denominators that are close to one another, they are attending to precision.

Adding a context to the problem above and following it up with a thought question, we now have a more rigorous task:

> Jessica and Lisa want to make bows. Jessica makes a bow that is 3/5 yard long, and Lisa makes a bow that is 7/10 yard long. Whose bow is longer? How do you know?

MP2: When students are able to accurately reason about the relative size of two quantities and justify how they know which is greater, they are exhibiting use of this practice.

MP6: When students are comparing two fractions with unlike denominators that are close to one another, they are attending to precision.

Bringing Clarity to Mathematical Practices via Task Analysis

Now, additional components are included in the previous task to raise it to a much higher level of cognitive demand: 1) A total amount of ribbon for each girl (not evenly divisible by both lengths) is included, and the context changes to have them consider making as many bows as possible, 2) justification of which girl makes the most bows, and 3) asking for a visual model to be included to represent the measurements.

Jessica and Lisa want to make bows. Each girl has 3 $\frac{1}{2}$ yards of ribbon. Jessica wants to use exactly 3/10 yard of ribbon in each of her bows. Lisa wants to use exactly 7/10 yard of ribbon in each of her bows.

a. How many bows can each girl make if she uses as much of the ribbon as possible?

b. Jessica says that she can make the best use of the ribbon because she will have the least ribbon leftover. Lisa says that she can make the best use of the ribbon because she will have the least ribbon leftover. Which girl is correct?

Draw a model to represent each girl's measurements and explain how you arrived at you answers to the questions above.

- MP1: When students are reading a word problem and deciding what operation should be used to solve the problem, they are making sense of the problem.
- MP2: When students are able to accurately reason about the relative size of two quantities and justify how they know which is greater, they are exhibiting use of this practice.
- MP3: When students can explain their argument about which student made the best use of the ribbon by considering both girls' thinking, they are constructing viable arguments and critiquing the reasoning of others.
- MP4: When students draw a picture to represent the problem and use it to justify their calculations, they are exhibiting use of models.
- MP6: When students are analyzing fractions with unlike denominators and considering leftover amounts, they are attending to precision as they correctly express the appropriate solution.
- MP7: When students are able to make use of numerical calculations or quantities involved in one girl's bows and apply it to the other girl's or if they are able to observe a pattern from their visual model and can extend that pattern over the scope of an entire problem, they are exhibiting use of this practice.

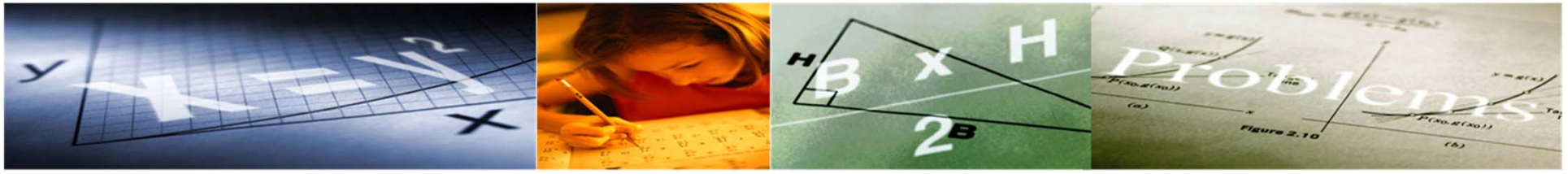
Connecting Mathematical Practice and Content Standards

"Not all tasks are created equal, and ***different tasks will provoke different levels and kinds of student thinking."***

Stein, Smith, Henningsen, & Silver, 2000

"***The level and kind of thinking in which students engage determines what they will learn."***

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997



Resources……..

- Progressions: narratives of the standards that describe how student skills and understanding in a particular domain develop from grade to grade http://ime.math.arizona.edu/progressions
- Illustrative Mathematics: illustrates the range and types of work that students will experience in a faithful implementation of the CCSS http://illustrativemathematics.org
- Common Core Tools: a blog moderated by Dr. William McCallum, lead for CCSS for Mathematics. Additional tools as they are developed are posted from time to time http://commoncoretools.wordpress.com