Implementing the Common Core State Standards for Mathematics

A Focus on Algebraic Mathematical Practices

Bowen Kerins

MassMATE, May 23, 2012

edc.org/cme/mpi

EDC
Education Development Center, Inc.

Question 1

Paul has all his marbles. There are p of them. Bowen has 7 fewer than Paul. How many marbles does Bowen have?

QUESTION 2

So there's this number. You can:

- add 4 to it...
- subtract 4 from it...
- multiply it by 4...
- divide it by 4...

And all four answers add up to exactly 60. What is the number?

Question 3

A. Is $(66, 43)$ on the line through $(7, 3)$ and $(10, 5)$?

Question 3

A. Is $(66, 43)$ on the line through $(7, 3)$ and $(10, 5)$?

B. Is $(107, 68)$ on the line through $(7, 3)$ and $(10, 5)$?

Question 3

A. Is $(66, 43)$ on the line through $(7, 3)$ and $(10, 5)$?

B. Is $(107, 68)$ on the line through $(7, 3)$ and $(10, 5)$?

C. What's an equation of the line through $(7, 3)$ and $(10, 5)$?

QUESTIONS, REDUX

1. Paul has all his marbles. There are p of them. Bowen has 7 fewer than Paul. How many marbles does Bowen have?
2. So there's this number. You can:
   - add 4 to it...
   - subtract 4 from it...
   - multiply it by 4...
   - divide it by 4...

   And all four answers add up to exactly 60. What is the number?
3. A. Is $(66, 43)$ on the line through $(7, 3)$ and $(10, 5)$?
   B. Is $(107, 68)$ on the line through $(7, 3)$ and $(10, 5)$?
   C. What's an equation of the line through $(7, 3)$ and $(10, 5)$?

What mathematicians most wanted and needed from me was to learn my ways of thinking, and not in fact to learn my proof of the geometrization conjecture for Haken manifolds.

—William Thurston

On Proof and Progress in Mathematics

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise. . . . Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

—CCSSM, p. 8 (2010)

Eight attributes of mathematical proficiency:

1. **Make sense of complex problems and persevere in solving them.**
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals . . . They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

Eight attributes of mathematical proficiency:

1. **Make sense of complex problems and persevere in solving them.**
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals . . . They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

   One CCSSM author says this is "the art of giving students problems to solve that they haven't been taught how to solve."

2. **Reason abstractly and quantitatively.**
   Mathematically proficient students make sense of the quantities and their relationships in problem situations. . . . Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize. . . and the ability to contextualize. . . .

2. **Reason abstractly and quantitatively.**
   Mathematically proficient students make sense of the quantities and their relationships in problem situations. . . . Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize. . . and the ability to contextualize. . . .

   In Algebra 1, variables start as placeholders for numbers, and problems should refer back to the quantities represented. Later, symbols become contexts in their own right, such as polynomial multiplication used to model probability problems. Tying calculations back to contexts is always relevant.

3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. . . .

3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. . . .

   The roles of proof and justification are not limited to a geometry course. Students should read and understand arguments while learning to construct them. Proofs and methods of construction should appear in algebra and geometry. Problems should ask students to critique and repair common errors in reasoning, or to read and analyze an argument between students.

4. **Model with mathematics.**
   Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

4. **Model with mathematics.**
   Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

   Students build mathematical models using functions, equations, graphs, tables, and technology. They also use mathematics to build approximate models for data, including a development of how the line of best fit is calculated. Students decide when a model is appropriate and determine limitations.

5. **Use appropriate tools strategically.**
   Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. . . .

5. **Use appropriate tools strategically.**
   Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. . . .

   Curricula should integrates tools and technology throughout. Students use technology to experiment with mathematical objects, to make tractable and enhance historically technical topics, and to build models of mathematical objects (both algebraic and geometric).

6. **Attend to precision.**
   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. ...

6. **Attend to precision.**
   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. . . .

   Definitions can be capstones. Students work toward greater precision in definitions as they move through a topic, even between courses. Students read dialogues or watch videos about other students wrestling with precise language. Students learn to critique definitions by looking for counterexamples, testing to see if the definition captures what they want it to.

7. **Look for and make use of structure.**
   Mathematically proficient students look closely to discern a pattern or structure. . . . For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

7. **Look for and make use of structure.**
Mathematically proficient students look closely to discern a pattern or structure. . . . For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Students look for hidden structure in arithmetic tables, and "chunk" portions of equations and expressions to explore their properties. Students transform area formulas to look for new meanings and connections.

8. **Look for and express regularity in repeated reasoning.** By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 1)/(x - 2) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and$(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series.

8. **Look for and express regularity in repeated reasoning.** By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 1)/(x - 2) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and$(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series.

   This is what the opening problems have in common. This practice is a general-purpose habit that will serve students well in all their technical endeavors. Adults use this behavior all the time. Let's dig deeper...

1. Students have trouble expressing generality with algebraic notation.
2. This is especially prevalent when they have to set up equations to solve word problems.
3. Many students have difficulty with slope, graphing lines, and finding equations of lines.
4. Building and using algebraic functions is another place where students struggle.

1. Students have trouble expressing generality with algebraic notation.
2. This is especially prevalent when they have to set up equations to solve word problems.
3. Many students have difficulty with slope, graphing lines, and finding equations of lines.
4. Building and using algebraic functions is another place where students struggle.

This list looks like a collection of disparate topics—using notation, solving word problems, ...

But if one looks underneath the topics to the mathematical habits that would help students master them, one finds a remarkable similarity:

A key ingredient in such a mastery is the reasoning habit of looking for and expressing regularity in repeated reasoning.

- This habit manifests itself when one is performing the same calculation over and over and begins to notice the “rhythm” in the operations.

But if one looks underneath the topics to the mathematical habits that would help students master them, one finds a remarkable similarity:

A key ingredient in such a mastery is the reasoning habit of looking for and expressing regularity in repeated reasoning.

- This habit manifests itself when one is performing the same calculation over and over and begins to notice the "rhythm" in the operations.
- Articulating this regularity leads to a generic algorithm, typically expressed with algebraic symbolism, that can be applied to any instance and that can be transformed to reveal additional meaning, often leading to a solution of the problem at hand.

Example A: The Dreaded Algebra Word Problem

Think about how hard it is for students to set up an equation that can be used to solve an algebra word problem. Some reasons for the difficulties include reading levels and unfamiliar contexts. But there has to be more to it than these surface features.

Consider, for example, the following two problems.

Example A: The Dreaded Algebra Word Problem

1. Nancy drives from Boston to Chicago at an average speed of 50mph and returns at an average speed of 60mph. The driving distance from Boston to Chicago is is 990 miles. For how many hours is Nancy on the road?

2. Nancy drives from Boston to Chicago at an average speed of 50mph and returns at an average speed of 60mph. Nancy is on the road for 36 hours. What is the driving distance from Boston to Chicago?

Example A: The Dreaded Algebra Word Problem

1. Nancy drives from Boston to Chicago at an average speed of 50mph and returns at an average speed of 60mph. The driving distance from Boston to Chicago is is 990 miles. For how many hours is Nancy on the road?

2. Nancy drives from Boston to Chicago at an average speed of 50mph and returns at an average speed of 60mph. Nancy is on the road for 36 hours. What is the driving distance from Boston to Chicago?

The problems have identical reading levels, and the context is the same in each. But teachers report that many students who can solve problem 1 are baffled by problem 2.

Example A: The Dreaded Word Problem

This is where the reasoning habit of "expressing the rhythm" in a calculation can be of great use. The basic idea:

- Guess at an answer to problem 2, and
- check your guess as if you were working on problem 1, *keeping track of your steps*.

Example A: The Dreaded Word Problem

This is where the reasoning habit of "expressing the rhythm" in a calculation can be of great use. The basic idea:

- Guess at an answer to problem 2, and
- check your guess as if you were working on problem 1, *keeping track of your steps*.

The purpose of the guess is not to stumble on (or to approximate) the correct answer; rather, it is to help you construct a "checking algorithm" that will work for any guess.

EDC
Education Development Center, Inc.

Problem 2: Nancy drives over at an average speed of 50mph and returns at an average of 60mph. He's on the road for 36 hours. What is the driving distance?

So, you take several guesses until you are able to express your checking algorithm in algebraic symbols. For example:

Example A: The Dreaded Word Problem

Problem 2: Nancy drives over at an average speed of 50mph and returns at an average of 60mph. He's on the road for 36 hours. What is the driving distance?

So, you take several guesses until you are able to express your checking algorithm in algebraic symbols. For example:

- Suppose the distance is 1000 miles.

Example A: The Dreaded Word Problem

Problem 2: Nancy drives over at an average speed of 50mph and returns at an average of 60mph. He's on the road for 36 hours. What is the driving distance?

So, you take several guesses until you are able to express your checking algorithm in algebraic symbols. For example:

- Suppose the distance is 1000 miles.
- How do I check the guess of 1000 miles? I divide 1000 by 50. Then I divide 1000 by 60. Then I add my answers together, to see if I get 36.

EXAMPLE A: THE DREADED WORD PROBLEM

Problem 2: Nancy drives over at an average speed of 50mph and returns at an average of 60mph. He's on the road for 36 hours. What is the driving distance?

So, you take several guesses until you are able to express your checking algorithm in algebraic symbols. For example:

- Suppose the distance is 1000 miles.
- How do I check the guess of 1000 miles? I divide 1000 by 50. Then I divide 1000 by 60. Then I add my answers together, to see if I get 36. I don't.

Education Development Center, Inc.

Example A: The Dreaded Word Problem

Problem 2: Nancy drives over at an average speed of 50mph and returns at an average of 60mph. He's on the road for 36 hours. What is the driving distance?

So, you take several guesses until you are able to express your checking algorithm in algebraic symbols. For example:

- Suppose the distance is 1000 miles.
- How do I check the guess of 1000 miles? I divide 1000 by 50. Then I divide 1000 by 60. Then I add my answers together, to see if I get 36. I don't.
- So, check another number—say 950. 950 divided by 50 plus 950 divided by 60. Is that 36?

Example A: The Dreaded Word Problem

Problem 2: Nancy drives over at an average speed of 50mph and returns at an average of 60mph. He's on the road for 36 hours. What is the driving distance?

So, you take several guesses until you are able to express your checking algorithm in algebraic symbols. For example:

- Suppose the distance is 1000 miles.
- How do I check the guess of 1000 miles? I divide 1000 by 50. Then I divide 1000 by 60. Then I add my answers together, to see if I get 36. I don't.
- So, check another number—say 950. 950 divided by 50 plus 950 divided by 60. Is that 36?
- No, but a general method is evolving that will allow me to check *any* guess.

EXAMPLE A: THE DREADED WORD PROBLEM

- My guess-checker is

$$\frac{\text{guess}}{50} + \frac{\text{guess}}{60} \stackrel{?}{=} 36$$

- So my *equation* is

$$\frac{\text{guess}}{50} + \frac{\text{guess}}{60} = 36$$

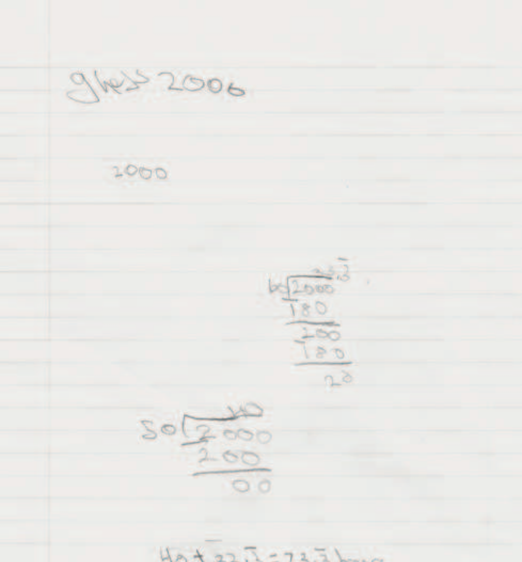
or, letting x stand for the unknown correct guess,

$$\frac{x}{50} + \frac{x}{60} = 36$$

EDC
Education Development Center, Inc.

Example A: The Dreaded Word Problem

Here’s some student work that shows how the process develops:



Example A: The Dreaded Word Problem

$40 + 33.\overline{3} = 73.\overline{3}$ hours

guess: 1500 mph

$1500 \div 60 = 25$

$$\begin{array}{r} 25 \\ 60\,\overline{)\,1500} \\ -120 \\ \hline 300 \\ -300 \\ \hline 0 \end{array}$$

25

$$\begin{array}{r} 30 \\ 50\,\overline{)\,1500} \\ -150 \\ \hline 00 \end{array}$$

$25 + 30 = 55$ hrs

$((\text{guess}) \div 60) + (\text{guess} \div 50) = 36$

$(x \div 60) + (x \div 50) = 36$

Example B: Equations for lines

The phenomenon was first noticed in precalculus . . .

The phenomenon was first noticed in precalculus ...
Graph

$$16x^2 - 96x + 25y^2 - 100y - 156 = 0$$

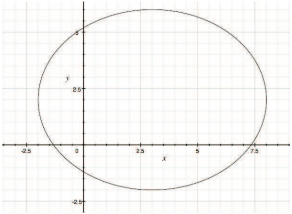
EXAMPLE B: EQUATIONS FOR LINES

The phenomenon was first noticed in precalculus ...
Graph

$$16\,x^2 - 96\,x + 25\,y^2 - 100\,y - 156 = 0$$

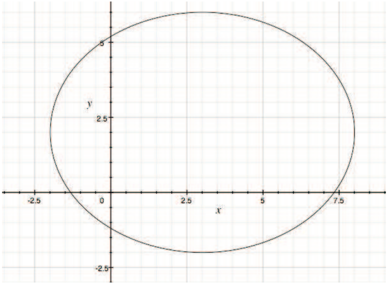
$$16\,x^2 - 96\,x + 25\,y^2 - 100\,y - 156 = 0 \Rightarrow \frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1 \Rightarrow$$



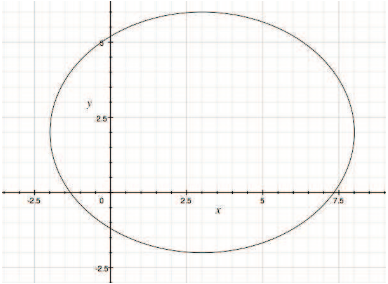
Example B: Equations for lines

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$



Example B: Equations for lines

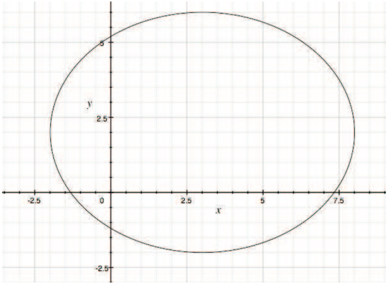
$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$



Is $(7.5, 3.75)$ on the graph?

EXAMPLE B: EQUATIONS FOR LINES

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$



Is $(7.5, 3.75)$ on the graph?

This led to the idea that “equations are point testers.”

Education Development Center, Inc.

Example B: Equations for lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.

Education Development Center, Inc.

Example B: Equations for lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.
- Students can draw the line, and, just as in the word problem example, they can guess at some points and check to see if they are on ℓ.

EDC
Education Development Center, Inc.

Example B: Equations for lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.
- Students can draw the line, and, just as in the word problem example, they can guess at some points and check to see if they are on ℓ.
- Trying some points like $(5, 1)$, $(3, 4)$, $(2, 2)$, and $(5, 17)$ leads to a generic guess-checker:

  To see if a point is on ℓ, you check that its x-coordinate is 5.

Example B: Equations for lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.
- Students can draw the line, and, just as in the word problem example, they can guess at some points and check to see if they are on ℓ.
- Trying some points like $(5, 1)$, $(3, 4)$, $(2, 2)$, and $(5, 17)$ leads to a generic guess-checker:

  To see if a point is on ℓ, you check that its x-coordinate is 5.

- This leads to a guess-checker: $x \stackrel{?}{=} 5$ and the equation

$$x = 5$$

- What about lines for which there is no simple guess-checker? The idea is to find a geometric characterization of such a line and then to develop a guess-checker based on that characterization. One such characterization uses *slope*.

Example B: Equations for lines

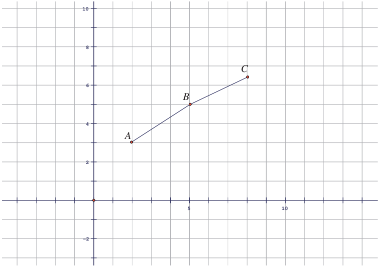
- What about lines for which there is no simple guess-checker? The idea is to find a geometric characterization of such a line and then to develop a guess-checker based on that characterization. One such characterization uses *slope*.

- In first-year algebra, students study slope, and one fact about slope that often comes up is that three points on the coordinate plane, not all on the same vertical line, are collinear if and only if the slope between any two of them is the same.

Example B: Equations for lines

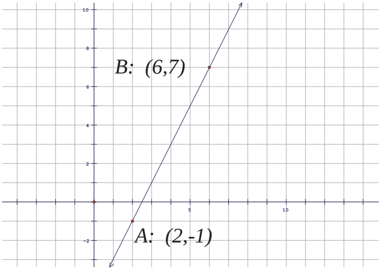
If we let $m(A,B)$ denote the slope between A and B (calculated as change in y-height divided by change in x-run), then the collinearity condition can be stated like this:

Basic assumption: A, B, and C are collinear $\Leftrightarrow m(A,B) = m(B,C)$



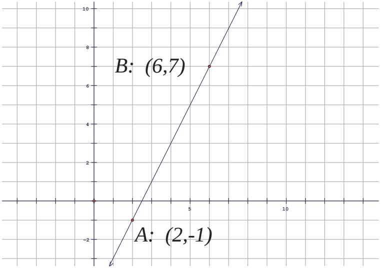
EXAMPLE B: EQUATIONS FOR LINES

What is an equation for $\ell = \overleftrightarrow{AB}$ if $A = (2, -1)$ and $B = (6, 7)$?



Example B: Equations for lines

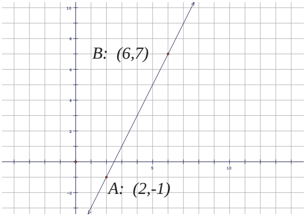
What is an equation for $\ell = \overleftrightarrow{AB}$ if $A = (2, -1)$ and $B = (6, 7)$?



Try some points, keeping track of the steps…

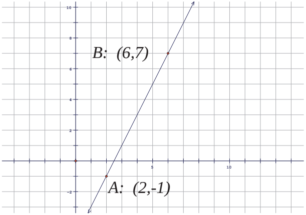
Example B: Equations for lines

- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$



Example B: Equations for lines

- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$

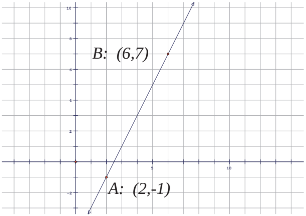


- Test $C = (3, 4)$:

  $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope

Example B: Equations for lines

- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$

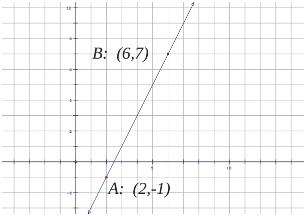


- Test $C = (3, 4)$: $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope
- Test $D = (5, 5)$: $m(D, B) = \frac{5-7}{5-6} \stackrel{?}{=} 2 \Rightarrow$ Yup

EDC
Education Development Center, Inc.

Example B: Equations for lines

- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$

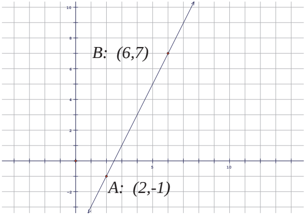


- Test $C = (3, 4)$:
  $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope
- Test $D = (5, 5)$:
  $m(D, B) = \frac{5-7}{5-6} \stackrel{?}{=} 2 \Rightarrow$ Yup
- The “guess-checker?”
  Test $P = (x, y)$:
  $m(P, B) = \frac{y-7}{x-6} \stackrel{?}{=} 2$

Education Development Center, Inc.

Example B: Equations for lines

- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$



- Test $C = (3, 4)$:
  $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope
- Test $D = (5, 5)$:
  $m(D, B) = \frac{5-7}{5-6} \stackrel{?}{=} 2 \Rightarrow$ Yup
- The “guess-checker?”
  Test $P = (x, y)$:
  $m(P, B) = \frac{y-7}{x-6} \stackrel{?}{=} 2$

And an equation is $\frac{y-7}{x-6} = 2$

Education Development Center, Inc.

Other Examples Where This Habit Is Useful

- Finding lines of best fit
- Building expressions ("*three less than a number*")
- Fitting functions to tables of data
- Deriving the quadratic formula
- Establishing identities in Pascal's triangle
- Using recursive definitions in a CAS or spreadsheet

⋮

Education Development Center, Inc.

"Factoring Pattern for $x^2 + bx + c$, c Negative"

“Factoring Pattern for $x^2 + bx + c$, c Negative”

Factor. Check by multiplying factors. If the polynomial is not factorable, write “prime.”

"Factoring Pattern for $x^2 + bx + c$, c Negative"

Factor. Check by multiplying factors. If the polynomial is not factorable, write "prime."

1. $a^2 + 4a - 5$	**2.** $x^2 - 2x - 3$	**3.** $y^2 - 5y - 6$
4. $b^2 + 2b - 15$	**5.** $c^2 - 11c - 10$	**6.** $r^2 - 16r - 28$
7. $x^2 - 6x - 18$	**8.** $y^2 - 10c - 24$	**9.** $a^2 + 2a - 35$
10. $k^2 - 2k - 20$	**11.** $z^2 + 5z - 36$	**12.** $r^2 - 3r - 40$
13. $p^2 - 4p - 21$	**14.** $a^2 + 3a - 54$	**15.** $y^2 - 5y - 30$
16. $z^2 - z - 72$	**17.** $a^2 - ab - 30b^2$	**18.** $k^2 - 11kd - 60d^2$
19. $p^2 - 5pq - 50q^2$	**20.** $a^2 - 4ab - 77b^2$	**21.** $y^2 - 2yz - 3z^2$
22. $s^2 + 14st - 72t^2$	**23.** $x^2 - 9xy - 22y^2$	**24.** $p^2 - pq - 72q^2$

To factor a trinomial of the form $ax^2 + bx + c$ where $a > 0$, follow these steps:

To factor a trinomial of the form $ax^2 + bx + c$ where $a > 0$, follow these steps:

A	B	C
F	H	D
G	I	E

To factor a trinomial of the form $ax^2 + bx + c$ where $a > 0$, follow these steps:

A	B	C
F	H	D
G	I	E

1. Identify the values of a, b, c. Put a in Box A and c in Box B. Put the product of a and c in Box C.
2. List the factors of the number from Box C and identify the pair whose sum is b. Put the two factors you find in Box D and E.
3. Find the greatest common factor of Boxes A [sic] and E and put it in box G.

Education Development Center, Inc.

From a published text (2010)

A	B	C
F	H	D
G	I	E

4 In Box F, place the number you multiply by Box G to get Box A.

5 In Box H, place the number you multiply by Box F to get Box D.

6 In Box I, place the number you multiply by Box G to get Box E.

Solution: The binomial factors whose product gives the trinomial are $(Fx + I)(Gx + H)$.

EDC
Education Development Center, Inc.

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

so...

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

so...

Find two numbers whose sum is 14 and whose product is 48.

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

so...

Find two numbers whose sum is 14 and whose product is 48.

$$(x + 6)(x + 8)$$

What about this one?

$$49x^2 + 35x + 6$$

What about this one?

$$49x^2 + 35x + 6$$

$$49x^2 + 35x + 6 = (7x)^2 + 5(7x) + 6$$

What about this one?

$$49x^2 + 35x + 6$$

$$\begin{aligned} 49x^2 + 35x + 6 &= (7x)^2 + 5(7x) + 6 \\ &= \clubsuit^2 + 5\clubsuit + 6 \end{aligned}$$

Education Development Center, Inc.

What about this one?

$$49x^2 + 35x + 6$$

$$\begin{aligned} 49x^2 + 35x + 6 &= (7x)^2 + 5(7x) + 6 \\ &= \clubsuit^2 + 5\clubsuit + 6 \\ &= (\clubsuit + 3)(\clubsuit + 2) \end{aligned}$$

What about this one?

$$49x^2 + 35x + 6$$

$$\begin{aligned} 49x^2 + 35x + 6 &= (7x)^2 + 5(7x) + 6 \\ &= \clubsuit^2 + 5\clubsuit + 6 \\ &= (\clubsuit + 3)(\clubsuit + 2) \\ &= (7x + 3)(7x + 2) \end{aligned}$$

What about this one?

$$6x^2 + 31x + 35$$

What about this one?

$$6x^2 + 31x + 35$$

$$6(6x^2 + 31x + 35) = (6x)^2 + 31(6x) + 210$$

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \end{aligned}$$

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned}6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210\\ &= \clubsuit^2 + 31\clubsuit + 210\\ &= (\clubsuit + 21)(\clubsuit + 10)\end{aligned}$$

Factoring Using The Structure Practice

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned}
6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\
&= \clubsuit^2 + 31\clubsuit + 210 \\
&= (\clubsuit + 21)(\clubsuit + 10) \\
&= (6x + 21)(6x + 10)
\end{aligned}$$

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned}
6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\
&= \clubsuit^2 + 31\clubsuit + 210 \\
&= (\clubsuit + 21)(\clubsuit + 10) \\
&= (6x + 21)(6x + 10) \\
&= 3(2x + 7) \cdot 2(3x + 5)
\end{aligned}$$

Factoring Using The Structure Practice

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned}
6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\
&= \clubsuit^2 + 31\clubsuit + 210 \\
&= (\clubsuit + 21)(\clubsuit + 10) \\
&= (6x + 21)(6x + 10) \\
&= 3(2x + 7) \cdot 2(3x + 5) \\
&= 6(2x + 7)(3x + 5)
\end{aligned}$$

EDC
Education Development Center, Inc.

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \\ &= 3(2x + 7) \cdot 2(3x + 5) \\ &= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots \end{aligned}$$

$$6(6x^2 + 31x + 35) = 6(2x + 7)(3x + 5)$$

EDC

Education Development Center, Inc.

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned}
6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\
&= \clubsuit^2 + 31\clubsuit + 210 \\
&= (\clubsuit + 21)(\clubsuit + 10) \\
&= (6x + 21)(6x + 10) \\
&= 3(2x + 7) \cdot 2(3x + 5) \\
&= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots
\end{aligned}$$

$$\cancel{6}(6x^2 + 31x + 35) = \cancel{6}(2x + 7)(3x + 5)$$

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \\ &= 3(2x + 7) \cdot 2(3x + 5) \\ &= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots \end{aligned}$$

$$6x^2 + 31x + 35 = (2x + 7)(3x + 5)$$

- This technique is perfectly general and can be used to transform a polynomial of any degree into one whose leading coefficient is 1.
- And it fits into the larger landscape of the *theory of equations* that shows how to use similar transformations to
  - remove terms
  - transform roots
  - derive "formulas" for equations of degree 3 and 4
  - extend the notion of *discriminant* to higher degrees

Other Examples Where Chunking Is Useful

- Completing the square and removing terms
- Solving trig equations
- Analyzing conics and other curves
- All over calculus
- Interpreting results from a computer algebra system

$$\vdots$$

For More Information

About MPI programs

Contact Melody Hachey:
mhachey@edc.org
`edc.org/cme/mpi`