Helping Teachers Transition to the New Standards for Mathematical Practice

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Agenda

- Review standard #3
- Examine four productive talk moves
- Summarize research findings about productive math talk
- Explore tasks that encourage productive math talk

Construct viable arguments and critique the reasoning of others.

- Mathematically proficient students …
  - justify their conclusions, communicate them to others, and respond to the arguments of others.
  - compare the effectiveness of two plausible arguments…and—if there is a flaw in an argument—explain what it is.
  - read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

What does all of this really mean?

- Students do *a lot of* talking about key ideas
- Student talk is used to help students learn (not to show what they have already learned)
- A.k.a. discourse-intensive instruction, mathematically productive talk, or math talk.

Construct viable arguments and critique the reasoning of others: An example

Lesson Topic: Comparing Fractions

- You, the teacher, pass out a worksheet with three pairs of fractions. You say, “Work with the people at your table to find the greater fraction in each pair. Be ready to explain your reasoning when we talk as a whole class.”
- Students work on the worksheet in groups of 3-4 students. You help as needed.
- The whole-class discussion begins.

Whole-Class Discussion on Comparing 2/5 and 3/4

Line	Speaker	Utterance
1	Teacher	Which is greater: 2/5 or 3/4? Alex?
2	Alex	I think ¾ is greater.
3	Teacher	Why do you think so?
4	Alex	Because I used a half.

Productive Talk Moves

What does the teacher do next after the student tells how he got his answer?

Press for Reasoning

You ask a student to deepen, clarify, or support his initial explanation.

Line	Speaker	Utterance
1	Teacher	Which is greater: 2/5 or 3/4? Alex?
2	Alex	I think ¾ is greater.
3	Teacher	Why do you think so?
4	Alex	Because I used a half.
5	Teacher	Okay, what do you mean 'you used a half'?

Press for Reasoning (cont.)

A <u>sustained</u> press for reasoning is often necessary.

Line	Speaker	Utterance
1	Teacher	Which is greater: 2/5 or 3/4? Alex?
2	Alex	I think ¾ is greater.
3	Teacher	Why do you think so?
4	Alex	Because I used a half.
5	Teacher	Okay, what do you mean 'you used a half'?
6	Alex	That thing we learned where we see if it's more or less than a half.

Press for Reasoning (cont.)

A <u>sustained</u> press for reasoning is often necessary.

Line	Speaker	Utterance
1	Teacher	Which is greater: 2/5 or 3/4? Alex?
2	Alex	I think ¾ is greater.
3	Teacher	Why do you think so?
4	Alex	Because I used a half.
5	Teacher	Okay, what do you mean 'you used a half'?
6	Alex	That thing we learned where we see if it's more or less than a half.
7	Teacher	Okay, so say more. How did you compare each fraction to one half?

Press for reasoning (cont.)

Line	Speaker	Utterance
6	Alex	That thing we learned where we see if it's more or less than a half.
7	Teacher	Okay, so say more. How did you compare each fraction to one half?
8	Alex	Well, for 2/5, I knew that half of 5 was 2 ½, and 2/5 is less than that. So 2/5 is less than ½.

Who can repeat?

You ask the other students to repeat or rephrase a peer's contribution.

Line	Speaker	Utterance
6	Alex	That thing we learned where we see if it's more or less than a half.
7	Teacher	Okay, so say more. How did you compare each fraction to one half?
8	Alex	Well, for 2/5, I knew that half of 5 was 2 ½, and 2/5 is less than that. So 2/5 is less than ½.
9	Teacher	Who can repeat what Alex just said? How did he know that 2/5 was less than ½? Maria?
10	Maria	He said that half would be 2 ½ over 5, but we only have 2 over 5, so that's less.

Turn-and-Talk

You ask everyone to "turn and talk to the person next to them" about one student's idea.

Line	Speaker	Utterance
8	Alex	Well, for 2/5, I knew that half of 5 was 2 ½, and 2/5 is less than that. So 2/5 is less than ½.
9	Teacher	Who can repeat what Alex just said? How did he know that 2/5 was less than ½? Maria?
10	Maria	He said that half would be 2 ½ over 5, but we only have 2 over 5, so that's less.
11	Teacher	This is an important but complex idea. Can everyone turn and talk to the person next to them about it. What did Alex do to find out that 2/5 was less than ½?

What do you think about that?

You ask students to endorse, refute, challenge, or add on to a classmate's idea.

Line	Speaker	Utterance
1	Teacher	Who used a different strategy? Katya?
2	Katya	Well, I just looked at the 2 and 3. Since 3 is greater than 2, ¾ has to be greater than 2/5.
3	Teacher	What do other people think about that?

What do you think about that? (cont.)

Line	Speaker	Utterance
1	Teacher	Who used a different strategy? Katya?
2	Katya	Well, I just looked at the 2 and 3. Since 3 is greater than 2, ¾ has to be greater than 2/5.
3	Teacher	What do other people think about that? Jan?
4	Jan	Well, yeah, it works but only 'cause 4ths are bigger than 5ths. So, 2/5 is two small pieces and ¾ is three bigger pieces. Having more of bigger pieces is better.

Do you agree or disagree.. and why?

Line	Speaker	Utterance
1	Teacher	Who used a different strategy? Katya?
2	Katya	Well, I just looked at the 2 and 3. Since 3 is greater than 2, ¾ has to be greater than 2/5.
3	Teacher	What do other people think about that? Jan?
4	Jan	Well, yeah, it works but only 'cause 4ths are bigger than 5ths. So, 2/5 is two small pieces and ¾ is three bigger pieces. Having more of bigger pieces is better.
5	Teacher	Okay, so you said a lot. Let's back up a little. The first thing you said was that 4ths are bigger than 5ths. Who agrees or disagrees with that statement?

Do you agree or disagree.. and why?

Line	Speaker	Utterance
1	Teacher	Who used a different strategy? Katya?
2	Katya	Well, I just looked at the 2 and 3. Since 3 is greater than 2, ¾ has to be greater than 2/5.
3	Teacher	What do other people think about that? Jan?
4	Jan	Well, yeah, it works but only 'cause 4ths are bigger than 5ths. So, 2/5 is two small pieces and ¾ is three bigger pieces. Having more of bigger pieces is better.
5	Teacher	Okay, so you said a lot. Let's back up a little. The first thing you said was that 4ths are bigger than 5ths. Who agrees or disagrees with that statement?
6	Louie	I agree.
7	Teacher	Why?
8	Louie	'Cause it's true.

Families of Talk Moves

- Press for reasoning
  - Say more about that.
  - What did you mean by [x]?
  - Can you say that again?
- Who can repeat?
  - Who can put that in their own words?
  - What else can say it again?
- What do you think about that?
  - Do you agree or disagree…and why?
  - Does that idea make sense to you?
  - Who can add on to that idea?

Frequently asked questions about using student talk in math class

Q: Does it really help students learn?

Q: Can middle grades students learn to talk this way?

Q: What about the struggling students?

"The good students give great explanations because they already know the answers. How does that help the students who can't talk because they don't know what to say?"

Math Talk & the Struggling Student

- Receiving explanations is not associated with achievement but…
  - Constructive activity after receiving an explanation ***is*** associated with achievement.
  - Specifically, (a) reworking the explanation from the start or (b) repeating it to a classmate are behaviors that are linked to achievement.
    - Sources: Webb, 1991; Webb & Mastergeorge, 2003; Webb, Troper, & Fall, 1995

Math Talk & the Struggling Student (cont.)

- In one study conducted at the 7th grade level,
  - students who reported that participation was a part of the process of learning math were more likely to give conceptual explanations,
  - those who reported that participation was "threatening" tended to talk about procedures only.
  - Students' ideas about politeness have an affect on how they participate.

    - Source: Jansen, A. (2008).

Q: Does this mean I have to stop teaching math directly to my students?

Q: What kinds of tasks support the practice of math talk?

A Typical Fraction Number Line Task

- Place the following set of fractions on the number line below. Be ready to explain how you knew where to place each number.

$$\frac{7}{8}, \frac{1}{4}, \frac{3}{5}, \frac{1}{12}$$

- Working with the person next to you, complete each of the following sentences…
  - This task may help students use productive math talk because…
  - This task may NOT help students use productive math talk because…

A Typical Fraction Number Line Task

This task may help students use productive math talk because…

- There are multiple solution strategies.

This task may NOT help students use productive math talk because…

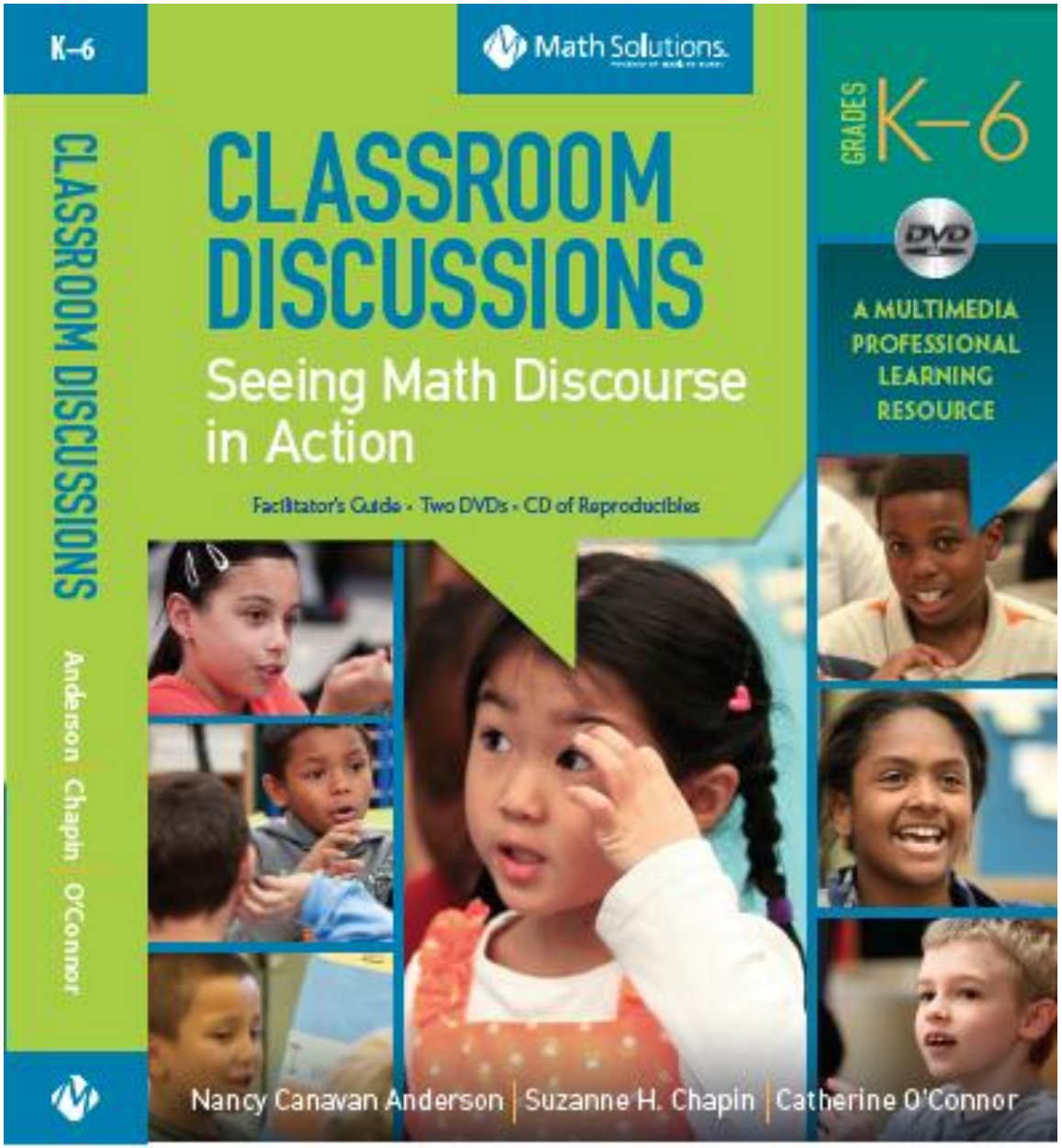
- There are multiple solution strategies.

Q: Can we modify the task so that (a) it still focuses on the same content standard and (b) allows for multiple solution strategies but (c) lends itself to productive but ***much more manageable*** classroom discussion?

An alternative fraction number line task

- Which is a better choice, 3/5 or 7/8, for the location marked A on the number line? Explain your reasoning.

- Source: Anderson, Chapin, O'Connor (2011).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Guidelines for Successful Video Showing

1. Assume that there are many things you don't know about the students, the classroom and the shared history of the teacher and students.
2. Assume good intent and expertise on the part of the teacher.
3. Focus on the following question, "What does the discuss reveal about student understanding?"

Source: Anderson, Chapin, O'Connor (2011).

Standard #6: Attend to precision.

- Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.
- They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
- In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Q: When my students talk in math class, their language, vocabulary, and explanations are often vague and imprecise. What do I do when they say something wrong?

Final Thoughts

- Productive math talk is messy.
- Small-group work time can be used to set-up the whole-class discussion.
- If a teacher doesn't understand a student's statement, encourage her to…
  - Persist and use *press for reasoning* several times in a row.
  - Use turn-and-talk as her own personal wait time.
- Remind teachers that they don't need to pursue every student's idea.

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