

**Braintree Math Crosswalk:
Kindergarten to Grade 5**

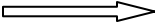
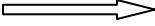
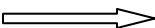
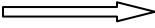
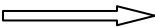
**The Massachusetts
2000 & 2004
Mathematics Frameworks
Compared with
The Massachusetts
2011
Mathematics Framework
Containing
Common Core
Mathematics Standards**

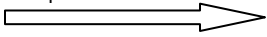
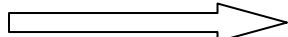
Kindergarten: Critical Areas

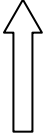
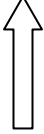
In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

- (1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in Kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.
- (2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

The Standards for Mathematical Practice complement the content standards at each grade level so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise.

<p style="text-align: center;">Kindergarten Massachusetts Math Standards</p>	<p style="text-align: center;">Kindergarten Common Core Standards</p>
<p>Number Sense & Operations</p>	
<p>Standard Changes</p> <p style="text-align: right;">    </p> <p>K.N.1 Match quantities up to at least 10 with numerals and words.</p> <p>K.N.2 Count by ones to at least 20.</p> <p style="text-align: right;">  </p>	<p>K.CC - Know number names and the count sequence.</p> <ol style="list-style-type: none"> 1. Count to 100 by ones and by tens. 2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment * Writes numbers to 20 beginning with 0.</p> </div> <ol style="list-style-type: none"> 4. Understand the relationship between numbers and quantities; connect counting to cardinality. <ol style="list-style-type: none"> a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment * Matches quantities to 20.</p> </div>
<p>K.N.3 Identify positions of objects in sequences (e.g., first, second) up to fifth.</p>	<p>Omitted</p>
<p>K.N.4 Compare sets of up to at least 10 concrete objects using appropriate language (e.g., none, more than, fewer than, same number of, one more than) and order numbers.</p> <p style="text-align: right;">  </p> <p>Standard Change</p>	<p>K.CC - Compare numbers.</p> <ol style="list-style-type: none"> 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. ¹ Include groups with up to ten objects 7. Compare two numbers between 1 and 10 presented as written numerals.

<p>Patterns, Relations & Functions</p>	
<p>K.P.1 Identify the attributes of objects as a foundation for sorting and classifying, e.g., a red truck, a red block, and a red ball share the attribute of being red; a square block, a square cracker, and a square book share the attribute of being square.</p> <p>K.P.2 Sort and classify objects by color, shape, size, number, and other properties</p>	<p>K.MD - Classify objects and count the number of objects in each category.</p> <p>3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.</p>
<p>K.P.3 Identify, reproduce, describe, extend, and create color, rhythmic, shape, number, and letter repeating patterns with simple attributes, e.g., ABABAB....</p>	<p>Omitted</p>
<p>K.P.4 Count by fives and tens at least up to 50.</p> <p>Standard Change </p>	<p>K.CC - Know number names and the count sequence.</p> <p>1. Count to 100 by ones and by tens.</p>
<p>Geometry</p>	
<p>K.G.1 Name, describe, sort, and draw simple two-dimensional shapes</p>	<p>K.G - Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).</p> <p>2. Correctly name shapes regardless of their orientations or overall size.</p> <p>3. Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid"). Analyze, compare, create, and compose shapes.</p>
<p>K.G.2 Describe attributes of two-dimensional shapes, e.g., number of sides, number of corners.</p> <p>K.G.3 Name and compare three-dimensional shapes.</p> <p>Standard Change </p>	<p>K.G Analyze, compare, create, and compose shapes.</p> <p>4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).</p> <p>5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.</p> <p>6. Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"</p>
<p>K.G.4 Identify positions of objects in space, and use appropriate language (e.g., beside, inside, next to, close to, above, below, apart) to describe and compare their relative positions.</p>	<p>K.G - Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).</p> <p>1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.</p>

<p>Measurement</p> <p>K.M.1 Recognize and compare the attributes of length, volume/capacity, weight, area, and time using appropriate language, e.g., longer, taller, shorter, same length; heavier, lighter, same weight; holds more, holds less, holds the same amount.</p> <p>K.M.3 Use nonstandard units to measure length, area, weight, and capacity.</p> <p>Standard change – no volume/capacity area and time</p> 	<p>K.MD - Describe and compare measurable attributes.</p> <p>1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.</p> <div style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment</p> <p>* Focus on measurable attributes such as length and weight.</p> </div> <p>2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.</p> <p>Standard change – teaching length and weight</p> 
<p>K.M.2 Make and use estimates of measurements from everyday experiences</p>	<p>Omitted</p> <div style="border: 1px dashed gray; padding: 5px;"> <p>Comment</p> <p>* Skill may be needed to learn other standards.</p> </div>
<p>Data Analysis, Statistics & Probability</p> <p>K.D.1 Collect, sort, organize, and draw conclusions about data using concrete objects, pictures, numbers, and graphs.</p>	<p>K.MD - Classify objects and count the number of objects in each category.</p> <p>3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.</p> <div style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment</p> <p>* Focus on categorizing and counting, not drawing conclusions.</p> </div>

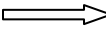

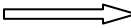
Grade 1: Critical Areas

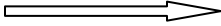
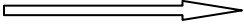
In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

- (1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
- (2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
- (3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.¹
- (4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

The Standards for Mathematical Practice complement the content standards at each grade level so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise.

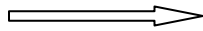
¹ Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Grade 1 Massachusetts Math Standards	Grade 1 Common Core Standards
<p>Number Sense & Operations</p> <p>1. N.1 Name and write (in numerals) whole numbers to 100, identify the place values of the digits, and order the numbers. </p> <p>Standard change from 100 to 120</p> <p>1.N.2 Identify cardinal numbers through 100 and ordinal numbers through 12.</p> <p>Standard change – no ordinal numbers </p>	<p>1.NBT Extend the counting sequence.</p> <p>1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</p> <p>1.NBT Understand place value.</p> <p>2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:</p> <p>A. 10 can be thought of as a bundle of ten ones — called a “ten.”</p> <p>B. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</p> <p>C. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</p> <div data-bbox="846 762 1471 905" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment</p> <p>* Addresses place value with specific reference to numbers between 11 and 19.</p> </div>
<p>1.N.3 Identify and represent common fractions (1/2, 1/3, 1/4) as parts of wholes, and parts of groups.</p> <p>Standard change – no 1/3 </p>	<p>1.G Reason with shapes and their attributes.</p> <p>3. Partition circles and rectangles into two and four equal shares, describe the shares using the words <i>halves</i>, <i>fourths</i>, and <i>quarters</i>, and use the phrases <i>half of</i>, <i>fourth of</i>, and <i>quarter of</i>. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.</p> <div data-bbox="870 1188 1495 1367" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment</p> <p>* Introduces the concept of half and quarter through concrete models and includes the decomposition of shapes.</p> </div>
<p>1.N.4 Compare whole numbers using terms and symbols, e.g., less than, equal to, greater than (<, =, >)</p>	<p>1.NBT Understand place value.</p> <p>3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.</p>
<p>1. N.5 Identify odd and even numbers and determine whether a set of objects has an odd or even number of elements</p>	<p>Omitted</p> <div data-bbox="870 1644 1495 1806" style="border: 1px dashed gray; padding: 5px;"> <p>Comment</p> <p>* Skill may be needed to learn other standards.</p> </div>

<p>1.N.6 Identify the value of pennies, nickels, dimes, quarters, and identify \$1, \$5, \$10, and \$20 bills. Find the value of a collection of coins to \$1.00</p> <p>Standard Change Know that coin identification is now being taught for the first time in grade 1. </p>	<p>1.MD Work with money MA.5 Identify the values of all U.S. coins; know their comparative values, e.g., a nickel is equivalent to 5 pennies. Use appropriate notations (e.g., 69 cents). Use the value of coins in the solution of problems.</p> <div style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comments * Additional Massachusetts Standard gives students experience with coins.</p> </div>
<p>1.N.7 Demonstrate an understanding with manipulatives of various meanings of addition and subtraction, e.g., addition as combination (plus, combined with, more); subtraction as comparison (how much less, how much more), equalizing (how many more are needed to make these equal), and separation (how much remaining).</p> <p>Standard change </p> <p>Addition of 3 whole numbers</p>	<p>1.OA Represent and solve problems involving addition and subtraction.</p> <p>1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</p> <div style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Focus on addition and subtraction within 20 and require using a symbol for an unknown number in an equation.</p> </div> <p>2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</p>
<p>1.N.8 Understand and use the inverse relationship between addition and subtraction (e.g., $8 + 6 = 14$ is equivalent to $14 - 6 = 8$) to solve problems and check solutions</p>	<p>1.OA Understand and apply properties of operations and the relationship between addition and subtraction.</p> <p>3. Apply properties of operations as strategies to add and subtract.² <i>Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)</i></p> <p>4. Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8. Add and subtract within 20.</p>

1. N.9 Know addition facts (addends to ten) and related subtraction facts with or without manipulatives, and use them to solve problems.

Standard Change



1.OA Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

DESE Comment

* Relates counting to addition and subtraction.

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

DESE Comments

* Requires addition and subtraction within 20 and fluency within 10. Also identifies multiple strategies. Requires automaticity of number facts in grade 2.

1.OA Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.

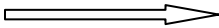
8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = _ - 3$, $6 + 6 = _$.

1.OA Work with addition and subtraction equations.

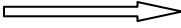
MA.9. Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20.

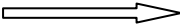
DESE Comments

* Additional Massachusetts Standard. Expects students to write and solve equations in problem situations.

<p>1. N.10 Demonstrate the ability to add and subtract two digit numbers accurately, without regrouping.</p> <p>1.N.11 Demonstrate in the classroom an understanding of and the ability to use the conventional algorithms for addition and subtraction without regrouping.</p> <p>Standard change- teach regrouping </p>	<p>1.NBT Use place value understanding and properties of operations to add and subtract.</p> <p>4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comments</p> <p>* Focuses on sense making strategies (based on place value, properties of operations, and/or relationship between addition and subtraction) for addition and does not include estimation.</p> </div>
<p>1. N.12 Estimate, calculate, and solve problems involving addition and subtraction of two-digit numbers. Describe differences between estimates and actual calculations</p>	<p>Omitted</p>
<p>Patterns, Relations, & Algebra</p>	
<p>1. P.1 Identify, reproduce, describe, extend, and create simple rhythmic, shape, size, number, color, and letter repeating patterns.</p>	<p>Omitted</p>
<p>2. P.2 Identify different patterns on the hundreds chart.</p>	<p>1.NBT Use place value understanding and properties of operations to add and subtract.</p> <p>5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</p> <p>6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>
<p>1. P.3 Describe and create addition and subtraction number patterns with manipulatives, e.g., 1, 4, 7, 10...: or 25, 23, 21...</p>	<p>Omitted</p>

<p>1. P.4 Skip count by twos, fives, and tens up to at least 100.</p>	<p>*Moved to Kindergarten Common Core Standard* K.CC - Know number names and the count sequence. Count to 100 by ones and by tens.</p> <p>*Moved to Grade 2 Common Core Standard* 2.NBT Understand place value. 2. Count within 1000; skip-count by 5s, 10s, and 100s.</p>
<p>1. P.5 Construct and solve open sentences that have variables, e.g., $_ + 7 = 10$ with manipulatives</p>	<p>1.OA Add and subtract within 20.</p> <p>5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</p> <p>6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).</p> <p>7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.</p> <p>8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = _ - 3$, $6 + 6 = _$.</p>
<p>1. P.6 Write number sentences using +, -, <, =, and/or > to represent mathematical relationships in everyday situations</p>	<p>1.NBT Understand place value.</p> <p>3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.</p>
<p>1. P.7 Describe functions related to trading, including coin trades with coins up to \$1.00</p>	<p>1.MD Work with money MA.5 Identify the values of all U.S. coins; know their comparative values, e.g., a nickel is equivalent to 5 pennies. Use appropriate notations (e.g., 69 cents). Use the value of coins in the solution of problems.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comments * Additional Massachusetts Standard gives students experience with coins.</p> </div>

<p>Geometry</p>	
<p>1.G.1 Describe attributes and parts of two & three dimensional shapes, e.g. number of corners, vertices, edges, and faces</p>	<p>1.G Reason with shapes and their attributes. 1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size) ; build and draw shapes to possess defining attributes.</p> <div style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Emphasizes distinction between defining and non-defining attributes.</p> </div>
<p>1. G.2 Identify and draw two-dimensional shapes, including both polygonal (up to four sides) and curved figures such as circles.</p> <p>Standard change </p> <p>Now teaching 3D shapes</p>	<p>1.G Reason with shapes and their attributes. 2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism."</p>
<p>1. G 3 Recognize congruent shapes.</p>	<p>Omitted</p>
<p>1. G.4 Identify shapes that have been rotated (turned), reflected (flipped), translated (slid), and enlarged.</p>	<p>Omitted</p>
<p>1. G.5 Identify line symmetry in two-dimensional shapes.</p>	<p>*Moved to Grade 4 Common Core Standard* 4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>
<p>1. G.6 Predict the results of putting shapes together and taking them apart.</p>	<p>1.G Reason with shapes and their attributes. 2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism."</p>
<p>1. G.7 Relate geometric ideas to numbers, e.g., seeing two rows as a model of addition.</p>	<p>Omitted</p>

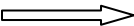
Measurement	
1. M.1 Identify parts of the day (e.g., morning, afternoon, evening), week, month, and calendar.	Omitted
1.M.2 Tell time hour and half hour on analog and digital clocks using am and pm.	1.MD Tell and write time. 3. Tell and write time in hours and half-hours using analog and digital clocks. <div style="border: 1px dashed gray; padding: 5px;"> DESE Comments * Does not include calendar intervals. </div>
1. M.3 Compare length and weight of two or more objects by using direct comparison. <p style="text-align: center;"> Standard Change - No weight  Just teaching length </p>	1.MD Measure lengths indirectly and by iterating length units. 1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. 2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. <i>Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.</i> <div style="border: 1px dashed gray; padding: 5px;"> DESE Comments * Focuses on measuring length and defines "length unit" as made up of shorter units. There is no mention of using proper tools. </div>
1. M.4 Measure and compare common objects using metric and English units of length measurement, e.g., centimeter, inch	Omitted
1. M.5 Select and correctly use the appropriate measurement tools, e.g., ruler, balance scale, thermometer	Omitted
Data Analysis, Statistics, & Probability	
1.D.1 Use interviews, surveys, and observations to gather data about themselves and their surroundings. 1.D.2 Organize, classify, represent, and interpret data using tallies, charts, tables, bar graphs, pictographs, and Venn diagrams; interpret the representations. 1.D.3 Formulate inferences (draw conclusions) and make educated guesses (conjectures) about a situation based on information gained from data.	1.MD Represent and interpret data. 4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. <div style="border: 1px dashed gray; padding: 5px;"> DESE Comment * Specifies types of questions to ask and answer. Does not require students to gather data. </div>

Grade 2: Critical Areas

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

- (1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
- (2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
- (3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
- (4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

The Standards for Mathematical Practice complement the content standards at each grade level so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise.

Grade 2 Massachusetts Math Standards	Grade 2 Common Core Standards
Number Sense & Operations	
<p>2.N.1 Name and write (in numerals) whole numbers to 1000, identify the place values of the digits, and order the numbers.</p> <p>2.N.2 Identify and distinguish among multiple uses of numbers, including cardinal (to tell how many) and ordinal (to tell which one) numbers, and numbers as labels and as measurements.</p>	<p>2.NBT Understand place value.</p> <p>2. Count within 1000; skip-count by 5s, 10s, and 100s.</p> <p>3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p> <div data-bbox="829 428 1453 527" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Requires counting within 1000.</p> </div>
<p>2.N.3 Identify and represent common fractions (1/2, 1/3, 1/4) as parts of wholes, parts of groups, and numbers on the number line</p> <p style="text-align: center;"></p> <p>Standard Change to Geometric representations</p>	<p>2.G Reason with shapes and their attributes.</p> <p>3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p> <div data-bbox="829 758 1453 919" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Focuses on quarters, thirds and halves in concrete models.</p> </div>
<p>2.N.4 Compare whole numbers using terms and symbols, e.g., less than, equal to, greater than (<, =, >)</p>	<p>2.NBT Understand place value.</p> <p>4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.</p> <div data-bbox="829 1083 1453 1182" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Limits comparisons to three-digit numbers.</p> </div>
<p>2.N.5 Identify odd and even numbers and determine whether a set of objects has an odd or even number of elements</p>	<p>2.OA Work with equal groups of objects to gain foundations for multiplication.</p> <p>3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</p> <div data-bbox="829 1474 1453 1587" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Includes even and odd.</p> </div>
<p>2.N.6 Identify the value of all US coins, and \$1, \$5, \$10, and \$20 bills Find the value of a collection of coins and dollar bills and different ways to represent an amount of money up to \$5 Use appropriate notation, e.g., 69¢, \$135</p>	<p>2.MD Work with time and money.</p> <p>8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?</p>

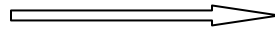
<p>2.N.7 Demonstrate an understanding of various meanings of addition and subtraction, e.g., addition as combination (plus, combined with, more); subtraction as comparison (how much less, how much more), equalizing (how many more are needed to make these equal), and separation (how much remaining)</p> <p>2.N.8 Understand and use the inverse relationship between addition and subtraction (e.g., $8 + 6 = 14$ is equivalent to $14 - 6 = 8$ and is also equivalent to $14 - 8 = 6$) to solve problems and check solutions</p>	<p>2.OA Represent and solve problems involving addition and subtraction.</p> <p>1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <div data-bbox="846 380 1471 596" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment</p> <p>* Includes two-step word problems and focuses on moving students toward more abstract representations of addition and multiplication.</p> </div>
<p>2.N.9 Know addition facts (addends to ten) and related subtraction facts, and use them to solve problems</p>	<p>2.OA Add and subtract within 20.</p> <p>2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.</p> <div data-bbox="846 804 1484 921" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment</p> <p>* Explicitly teach mental math strategies.</p> </div> <p>MA.2. By the end of grade 2, know from memory related subtraction facts of sums of two one-digit numbers.</p> <div data-bbox="846 1014 1484 1167" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comments</p> <p>* Additional Massachusetts standard specifies knowing subtraction facts.</p> </div>

2.N.10 Demonstrate the ability to add and subtract three-digit numbers accurately and efficiently.

2.N.11 Demonstrate in the classroom an understanding of and the ability to use the conventional algorithms for addition (two 3-digit numbers and three 2-digit numbers) and subtraction (two 3-digit numbers)

2.N.12 Estimate, calculate, and solve problems involving addition and subtraction of two-digit numbers. Describe differences between estimates and actual calculations.

Standard Change



2.NBT Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.

Understand the following as special cases:

A. 100 can be thought of as a bundle of ten tens — called a “hundred.”

B. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

6. **Add up to four two-digit numbers** using strategies based on place value and properties of operations.

DESE Comment

* Expects addition of up to four 2-digit numbers.

7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

DESE Comment

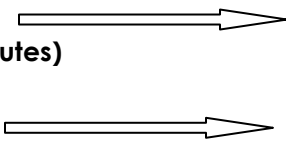
* Stresses the importance of using concrete models, drawings, and strategies based on place value to perform and understand addition and subtraction.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations.¹

¹ Explanations may be supported by drawings or objects.

DESE Comment

* Focuses on why strategies work for addition and subtraction.

Geometry	
<p>2.G.1 Describe attributes and parts of two- and three-dimensional shapes, e.g., length of sides, and number of corners, edges, faces, and sides.</p> <p>2.G.2 Identify, describe, draw, and compare two-dimensional shapes, including both polygonal (up to six sides) and curved figures such as circles.</p>	<p>2.G Reason with shapes and their attributes.</p> <p>1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.¹ Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. ¹ Sizes are compared directly or visually, not compared by measuring.</p>
<p>2.G.3 Recognize congruent shapes.</p>	<p>Omitted</p>
<p>2.G.4 Identify shapes that have been rotated (turned), reflected (flipped), translated (slid), and enlarged. Describe direction of translations, e.g., left, right, up, down.</p>	<p>Omitted</p>
<p>2.G.5 Identify line symmetry in two-dimensional shapes</p>	<p>*Moved to Grade 4 Common Core Standard*</p> <p>4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</p> <p>3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>
<p>2.G.6 Predict the results of putting shapes together and taking them apart.</p>	<p>2.G Reason with shapes and their attributes.</p> <p>2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.</p>
<p>2.G.7 Relate geometric ideas to numbers, e.g., seeing rows in an array as a model of repeated addition.</p>	<p>Omitted</p>
Measurement	
<p>2.M.1 Identify parts of the day (e.g., morning, afternoon, evening), days of the week, and months of the year. Identify dates using a calendar.</p> <p>2.M.2 Tell time at quarter-hour intervals on analog and digital clocks using am and pm</p> <p>Standard Change (nearest 5 minutes)</p> 	<p>2.MD Work with time and money.</p> <p>7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p> <p>MA.7a Know the relationships of time, including seconds in a minute; minutes in an hour; hours in a day; days in a week, month or year; weeks in a month or year.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment</p> <p>* Massachusetts additional standard requires knowing conversions related to time.</p> </div>

2.M.3 Compare the length, weight, area, and volume of two or more objects by using direct comparison

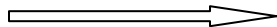
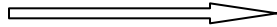
**Standard Change – weight/volume to grade 3
Area is taught as a geometry standard -2G2**



2.M.4 Measure and compare common objects using metric and English units of length measurement, e.g., centimeter, inch

2.M.5 Select and correctly use the appropriate measurement tools, e.g., ruler, balance scale, thermometer

2.M.6 Make and use estimates of measurement, including time, volume, weight, and area



2.MD Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

DESE Comment

* Focus on measuring length.

2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

3. Estimate lengths using units of inches, feet, centimeters, and meters.

4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

DESE Comment

* Requires expression of length difference in units.

2.MD Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

DESE Comment

* Includes word problems involving length.

6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

2.MD Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

Data Analysis, Statistics, & Probability	
<p>2.D.1 Use interviews, surveys, and observations to gather data about themselves and their surroundings</p> <p>2.D.2 Organize, classify, represent, and interpret data using tallies, charts, tables, bar graphs, pictographs, and Venn diagrams; interpret the representations</p> <p>2.D.3 Formulate inferences (draw conclusions) and make educated guesses (conjectures) about a situation based on information gained from data</p>	<p>2.MD Represent and interpret data.</p> <p>10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.</p> <div data-bbox="829 367 1453 489" style="border: 1px dashed black; padding: 5px;"> <p>DESE Comment * Focuses on picture and bar graphs.</p> </div>
<p>2.D.4 Decide which outcomes of experiments are most likely</p>	<p>Omitted</p>

Grade 3: Critical Areas

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

- (1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
- (2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, $\frac{1}{2}$ of the paint in a small bucket could be less paint than $\frac{1}{3}$ of the paint in a larger bucket, but $\frac{1}{3}$ of a ribbon is longer than $\frac{1}{5}$ of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
- (3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
- (4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

The Standards for Mathematical Practice complement the content standards at each grade level so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise.

<p style="text-align: center;">Grade 3 Massachusetts Math Standards</p>	<p style="text-align: center;">Grade 3 Common Core Standards</p>
<p>Number Sense & Operations</p>	
<p>3.N.1 Exhibit an understanding of the values of the digits in the base ten number system by reading, modeling, writing, comparing and ordering whole numbers through 9,999.</p> <p>3.N.2 Represent, order, and compare large numbers through 9,999. Represent numbers using expanded notation (e.g. $853 = 8 \times 100 + 5 \times 10 + 3$), and written out in words (e.g. eight hundred fifty-three).</p>	<p style="text-align: center;">*Moved to Grade 2 Common Core Standard*</p> <p>2.NBT Understand place value.</p> <p>3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p>

3.N.3 Identify and represent fractions (between 0 and 1 with denominators through 10) as parts of unit wholes and parts of groups. Model and represent a mixed number (with denominator 2, 3 or 4) as a whole number and a fraction (e.g. $1\frac{2}{3}$, $3\frac{1}{2}$).

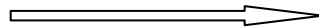


Standard Change

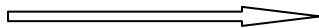
denominator is now 2,3,4,6,8

3.N.4 Locate on the number line and compare fractions (between 0 and 1 with denominators 2, 3, or 4; e.g. $\frac{2}{3}$).

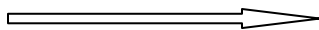
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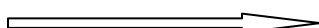
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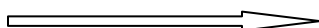
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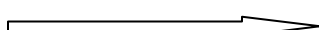
Standard Change



Standard Change



Standard Change



3. NF Develop understanding of fractions as numbers.

Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

1. Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
 - A. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

DESE Comment

* Note emphasis on unit fractions.

- B. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

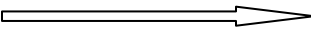
DESE Comment

* Introduce equivalence of fractions and reasoning about relative size.

3. **Explain equivalence of fractions** in special cases, and compare fractions by reasoning about their size.
 - A. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
 - B. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
 - C. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.*
 - D. Compare two fractions with the same **numerator** or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

3.G Reason with shapes and their attributes.

2. Partition shapes into parts with **equal areas**. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.*

<p>3.N.5 Recognize classes to which a number may belong (odd numbers, even numbers, and multiples of numbers through 10). Identify the numbers in those classes (e.g. the class of multiples of 7 between 1 and 29 consists of 7, 14, 21, 28).</p>	<p>* Moved to Grade 2 Common Core Standard* 2.OA Work with equal groups of objects to gain foundations for multiplication. 3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</p> <p>* Moved to Grade 4 Common Core Standard* 4.OA Gain familiarity with factors and multiples. 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p>
<p>3.N.6 Select, use, and explain various meanings and models of multiplication (through 10 x 10). Relate multiplication problems to corresponding division problems (e.g. draw a model to represent 5x6 and 30 divided by 6).</p> <p>3.N.7 Use the commutative (order) and identity properties of addition and multiplication on whole numbers in computations and problem situations, e.g. $3 + 4 + 7 = 3 + 7 + 4 = 10 + 4$.</p>	<p>3.OA Understand properties of multiplication and the relationship between multiplication and division. (Students need not use formal terms for these properties.) 5. Apply properties of operations as strategies to multiply and divide.² <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i></p> <p>DESE Comment * Includes associative and distributive properties.</p> <p>6. Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.</p> <p>DESE Comment * Approaches division as unknown factor problems.</p> <p>3.OA Multiply and divide within 100. 7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</p>
<p>3.N.8 Select and use appropriate operation's (addition, subtraction, multiplication, and division) to solve problems, including those involving money.</p> <p>Standard Change </p>	<p>3.OA Solve problems involving the four operations, and identify and explain patterns in arithmetic. 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Excludes multiplicative comparison problems (problems involving notions of "times as much"))</p>

3.N.9 Know multiplication facts through 10×10 and related division facts (e.g. $9 \times 8 = 72$ and 72 divided by 9 = 8). Use these facts to solve related problems and compute related problems (e.g., 3×5 is related to $30 \div 5$).

3.OA Represent and solve problems involving multiplication and division.

1. **Interpret** products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*

DESE Comment

* Interpret products through the relationship between objects in groups.

2. **Interpret** whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*

DESE Comment

* Interpret division as partitioning.

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations **with a symbol for the unknown** number to represent the problem.¹

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*

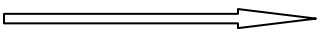
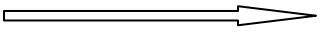
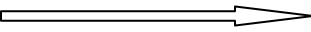
3.OA Understand properties of multiplication and the relationship between multiplication and division. (Students need not use formal terms for these properties.)

5. Apply properties of operations as strategies to multiply and divide.² *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

6. Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

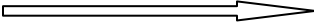
3.OA Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

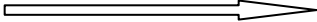
<p>3.N.10 Add and subtract (up to four-digit numbers) and multiply (up to two-digit numbers by a one-digit number) accurately and efficiently.</p> <p>Standard Change </p> <p>Standard Change </p>	<p>3.NBT Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)</p> <p>2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.</p>
<p>3.N.11 Round whole numbers through 1,000 to the nearest 10, 100, and 1,000.</p> <p>Standard Change </p> <p>No longer rounding to nearest 1000</p> <p>3.N.12 Understand and use the strategies of rounding and regrouping to estimate quantities, measures, and the results of whole-number computations (addition, subtraction, and multiplication) up to two-digit whole numbers and amounts of money to \$100, and to judge the reasonableness of the answer.</p>	<p>3.NBT Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)</p> <p>1. Use place value understanding to round whole numbers to the nearest 10 or 100.</p> <p>3.OA Solve problems involving the four operations, and identify and explain patterns in arithmetic.</p> <p>8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Excludes multiplicative comparison problems (problems involving notions of “times as much”))</p> <div style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Specifies two step word problems and their representation using equations, and includes order of operations.</p> </div>
<p>3.N.13 Use concrete objects and visual models to add and subtract (only when the answer is greater than or equal to zero) common fractions (halves, thirds, fourths, sixths, and eighths) with like denominators.</p>	<p>* Moved to Grade 4 Common Core Standard*</p> <p>4.NF Extend understanding of fraction equivalence and ordering. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100)</p> <p>A. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p>
<p>Patterns, Relations, & Algebra</p>	
<p>3.P.1 Create, describe, extend, and explain symbolic (geometric) patterns and addition and subtraction patterns (e.g. 2, 6, 10, ...; and 50, 45, 40....).</p>	<p>3.OA Solve problems involving the four operations, and identify and explain patterns in arithmetic.</p> <p>9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i></p> <div style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Requires explanation of patterns using properties of operations.</p> </div>

<p>3.P.2 Determine which symbol (>, <, or =) is appropriate for a given number sentence (e.g. $7 \times 8 \text{ ? } 49 + 6$).</p>	<p>Omitted</p>
<p>3.P.3 Determine values of variables (through 10) in simple equations involving addition, subtraction, or multiplication (e.g., $4106 - \tilde{N} = 37$; $c - m = 3$ and $4 + 5 = m + 3$).</p> <p>3.P.4 Write number sentences using +, -, x, <, =, > as well as the standard division symbol to represent mathematical relationships in everyday situations.</p>	<p>3.OA Represent and solve problems involving multiplication and division. (also listed above for 3.N.9)</p> <p>4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times \text{?} = 48$, $5 = _ \div 3$, $6 \times 6 = \text{?}$</i></p>
<p>Geometry</p>	
<p>3.G.1 Compare and analyze attributes and other features (e.g., number of sides, faces, corners, right angles, diagonals, and symmetry) of two-dimensional geometric shapes.</p> <p>3.G.2 Describe, model, draw, compare, and classify two-dimensional shapes, e.g., circles, triangles and quadrilaterals. Identify and describe simple three-dimensional shapes, e.g. cubes, spheres, and pyramids.</p>	<p>3.G Reason with shapes and their attributes.</p> <p>1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment * Includes nested categories.</p> </div>

<p>3.G.3 Identify angles as right angles, less than a right angle, and greater than a right angle.</p>	<p>* Moved to Grade 4 Common Core Standard* 4.MD Geometric measurement: understand concepts of angle and measure angles. 5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: A. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. B. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. 6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measurement.</p>
<p>3.G.4 Identify and draw intersecting, parallel, and perpendicular lines.</p>	<p>* Moved to Grade 4 Common Core Standard* 4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p>
<p>3.G.5 Using ordered pairs of whole numbers and/or letters locate and identify points on a grid.</p>	<p>* Moved to Grade 5 Common Core Standard* 5.G Graph points on the coordinate plane to solve real-world and mathematical problems. 1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond. 2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p>
<p>3.G.6 Identify and draw line symmetry in two-dimensional shapes.</p>	<p>* Moved to Grade 4 Common Core Standard* 4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>
<p>3.G.7 Predict and explain the results of taking apart and combining two-dimensional shapes.</p>	<p>3.G. Reason with shapes and their attributes 2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into 4 parts with equal area, and describe the area of each part is $\frac{1}{4}$ of the area of the shape.</i></p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>Comment * Debatable about how much of the old standard is in the new standard. New standard is much more about fractions.</p> </div>

Measurement	
<p>3.M.1 Demonstrate an understanding of such attributes as length, area, and weight, and select the appropriate type of unit for measuring each attribute using both U.S. customary (English) and metric measures.</p> <p>Standard Change </p>	<p>3.MD Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</p> <p>2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems (problems involving notions of “times as much”))</p> <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment * Does not specify using US customary units.</p> </div> <p>4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</p>
<p>3.M.2 Carry out simple unit conversions within a system of measurement, e.g., hours to minutes, cents to dollars, yards to feet or inches, etc.</p>	<p>Omitted but skills may be needed for word problems.</p>
<p>3.M.3 Identify time to the minute on analog and digital clocks using a.m. and p.m. Compute elapsed time using a clock less than one hour (e.g. minutes since...) and using a calendar (e.g., days since...).</p>	<p>3.MD Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</p> <p>1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p>
<p>3.M.4 Estimate and find area and perimeter of a rectangle using diagrams, models, and grids or by measuring.</p>	<p>3.MD Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</p> <p>5. Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>A. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment *Explicitly calls for learning square units.</p> </div> <p>B. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</p> <p>6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</p> <p>7. Relate area to the operations of multiplication and addition.</p> <p>A. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</p>

	<p>B. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>C. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.</p> <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment * Specifies using the distributive property using area models and recognizing area as additive.</p> </div> <p>D. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</p> <p>3.MD Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</p> <p>8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</p> <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment * Emphasizes connection between area and perimeter. Also includes all polygons.</p> </div>
<p>3.M.5 Identify and use appropriate metric and U.S. Customary (English) units and tools (e.g., ruler, scale, thermometer, and clock) to estimate, measure, and solve problems involving length, area, weight, time, and temperature.</p>	<p>3.MD Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</p> <p>2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems (problems involving notions of "times as much"))</p>

Data Analysis, Statistics, & Probability	
3.D.1 Collect and organize data using observations, measurements, surveys, or experiments, and identify appropriate ways to display the data.	Omitted
<p>3.D.2 Match representations of a data set in the forms of tables, line plots, pictographs, tallies, or bar graphs with the actual data set.</p> <p>3.D.3 Construct and draw conclusions from representations of data sets in the forms of tables, pictographs, line graphs, tallies, and bar graphs.</p>	<p>3.MD Represent and interpret data.</p> <p>3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i></p> <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment * Limits representations to scaled picture graphs and scaled bar graphs.</p> </div> <p>4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</p> <div style="border: 1px dashed gray; padding: 5px; margin: 10px 0;"> <p>DESE Comment * Note rulers marked in halves and fourths of an inch and horizontal scales in wholes, halves and quarters.</p> </div>
<p>Standard Change </p>	
3.D.4 List and count the number of possible outcomes of objects from two sets, e.g. how many different outfits can one make from a set of two sweaters and a set of three skirts.	Omitted

Grade 4: Critical Areas

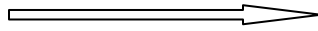
In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

- (1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
- (2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $15/9 = 5/3$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
- (3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

The Standards for Mathematical Practice complement the content standards at each grade level so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise.

Grade 4 Massachusetts Mathematics Standards	Grade 4 Common Core Standards
Number Sense & Operations	
<p>4.N.1 Exhibit an understanding of the base ten number system by reading, modeling, writing, and interpreting whole numbers to at least 100,000; demonstrating an understanding of the values of the digits; and comparing and ordering the numbers.</p> <p>4.N.2 Represent, order, and compare large numbers (to at least 100,000) using various forms, including expanded notation, e.g., $853 = 8 \times 100 + 5 \times 10 + 3$.</p>	<p>4.NBT Generalize place value understanding for multi-digit whole numbers.</p> <p>1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</i></p> <p>2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p> <p>3. Use place value understanding to round multi-digit whole numbers to any place.</p>
<p>4.N.3 Demonstrate an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on the number line.</p>	<p>* Moved to Common Core Grade 3 Standard*</p> <p>3. NF Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.</p> <p>1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p> <p>2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>A. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p>

Standard Change - also include 100 as a denominator



4.N.4 Select, use, and explain models to relate common fractions and mixed numbers ($1/2$, $1/3$, $1/4$, $1/5$, $1/6$, $1/8$, $1/10$, $1/12$, and $11/2$), find equivalent fractions, mixed numbers, and decimals, and order fractions.

4.NF Extend understanding of fraction equivalence and ordering. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100)

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

DESE Comment

* Focuses on comparisons of fractions that refer to the same whole and on justifying conclusions.

4.NF Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

A. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

B. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1/8 = 8/8 + 1/8$.

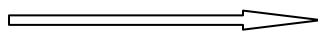
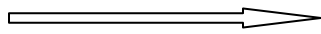
DESE Comment

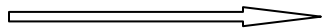
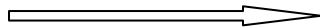
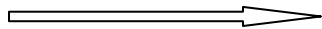
* Adds and subtracts fractions with like denominators.

C. **Add and subtract mixed numbers** with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

D. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Standard Changes





4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

A. Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

B. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)

C. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

4.N.5 Identify and generate equivalent forms of common decimals and fractions less than one whole (halves, quarters, fifths, and tenths).

4.N.6 Exhibit an understanding of the base ten number system by reading, naming, and writing decimals between 0 and 1 up to the hundredths.

4.NF Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.² For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)

DESE Comment

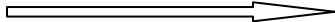
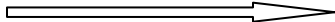
* Specifies working with fractions with denominators 10 and 100.

6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

DESE Comment

* Includes locating decimals on the number line and connecting decimals to the metric system.

	<p>7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment * Requires comparison of decimals that refer to the same whole and justifying conclusions.</p> </div>
<p>4.N.7 Recognize classes (in particular, odds, evens; factors or multiples of a given number; and squares) to which a number may belong, and identify the numbers in those classes. Use these in the solution of problems.</p>	<p>4.OA Gain familiarity with factors and multiples. 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment * Requires finding all factor pairs and includes prime and composite.</p> </div>
<p>4.N.8 Select, use, and explain various meanings and models of multiplication and division of whole numbers. Understand and use the inverse relationship between the two operations.</p>	<p>4.OA Use the four operations with whole numbers to solve problems. 1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment * Interprets multiplication as a comparison and explicitly requires representing statements as equations.</p> </div>
<p>4.N.9 Select, use, and explain the commutative, associative, and identity properties of operations on whole numbers in problem situations, e.g., $37 \times 46 = 46 \times 37$, $(5 \times 7) \times 2 = 5 \times (7 \times 2)$.</p>	<p style="text-align: center;">*Moved to Common Core Grade 3 Standard*</p> <p>3.OA Understand properties of multiplication and the relationship between multiplication and division. (Students need not use formal terms for these properties.)</p> <p>5. Apply properties of operations as strategies to multiply and divide.² <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i></p>

<p>4.N.10 Select and use an appropriate operation(s) (addition, subtraction, multiplication, and division) to solve problems, including those involving money.</p>	<p>4.OA Use the four operations with whole numbers to solve problems. 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p>
<p>4.N.11 Know multiplication facts through 12 x 12 and related division facts. Use these facts to solve related multiplication problems and compute related problems, e.g., 3 x 5 is related to 30 x 50, 300 x 5, and 30 x 500.</p>	<p>4.NBT Use place value understanding and properties of operations to perform multi-digit arithmetic. MA. 5A Know multiplication facts and related division facts through 12x12.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>Comment * This is a Massachusetts addition.</p> </div>
<p>4.N.12 Add and subtract (up to five-digit numbers) and multiply (up to three digits by two digits) accurately and efficiently.</p> <p>4.N.13 Divide up to a three-digit whole number with a single-digit divisor (with or without remainders) accurately and efficiently. Interpret any remainders.</p> <p>Standard Change </p> <p>4.N.16 Select and use a variety of strategies (e.g., front-end, rounding, and regrouping) to estimate quantities, measures, and the results of whole-number computations up to three-digit whole numbers and amounts of money to \$1000, and to judge the reasonableness of the answer.</p> <p>Standard Change </p>	<p>4.NBT Use place value understanding and properties of operations to perform multi-digit arithmetic. 4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment * Does not explicitly require the standard algorithm for multiplication.</p> </div> <p>6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>

<p>4.N.14 Demonstrate in the classroom an understanding of and the ability to use the conventional algorithms for addition and subtraction (up to five-digit numbers), and multiplication (up to three digits by two digits).</p> <p>4.N.15 Demonstrate in the classroom an understanding of and the ability to use the conventional algorithm for division of up to a three-digit whole number with a single-digit divisor (with or without remainders).</p>	<p>Omitted.</p> <div data-bbox="878 163 1503 300" style="border: 1px dashed gray; padding: 5px;"> <p>Comment * It was always vague what “demonstrate in the classroom” meant.</p> </div>
<p>4.N.17 Use concrete objects and visual models to add and subtract common fractions.</p>	<p>Aligns with Common Core Standard 4.NF about fractions.</p>
<p>4.N.18 Round whole numbers through 100,000 to the nearest 10, 100, 1000, 10,000, and 100,000.</p>	<p>4.OA Use the four operations with whole numbers to solve problems. 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <div data-bbox="834 848 1459 951" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Explicitly requires multistep word problems.</p> </div>

Patterns, Relations & Algebra	
<p>4.P.1 Create, describe, extend, and explain symbolic (geometric) and numeric patterns, including multiplication patterns like 3, 30, 300, 3000,</p>	<p>4.OA Generate and analyze patterns. 5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way</i></p>
<p>4.P.2 Use symbol and letter variables (e.g., D, x) to represent unknowns or quantities that vary in expressions and in equations or inequalities (mathematical sentences that use =, <, >).</p> <p>4.P.3 Determine values of variables in simple equations, e.g., $4106 - \tilde{N} = 37$; $c - m = 3$ and $4 + 5 = m + 3$.</p> <p>4.P.4 Use pictures, models, tables, charts, graphs, words, number sentences, and mathematical notations to interpret mathematical relationships.</p>	<p>4.OA Use the four operations with whole numbers to solve problems. 2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</p>
<p>4.P.5 Solve problems involving proportional relationships, including unit pricing (e.g., four apples cost 80¢, so one apple costs 20¢) and map interpretation (e.g., one inch represents five miles, so two inches represent ten miles).</p>	<p>*Moved to Common Core Grade 7 Standard*</p>
<p>4.P.6 Determine how change in one variable relates to a change in a second variable, e.g., input-output tables.</p>	<p>Omitted but relates to 4 OA 5</p>

<p>4.G.5 Describe and draw intersecting, parallel, and perpendicular lines.</p>	<p>4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment * Includes rays and line segments.</p> </div>
<p>4.G.6 Using ordered pairs of numbers and/or letters, graph, locate, identify points, and describe paths (first quadrant).</p>	<p style="text-align: center;">*Moved to Common Core Grade 5 Standard*</p> <p>5.G Graph points on the coordinate plane to solve real-world and mathematical problems. 1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</p>
<p>4.G.7 Describe and apply techniques such as reflections (flips), rotations (turns), and translations (slides) for determining if two shapes are congruent.</p>	<p style="text-align: center;">*Moved to Common Core Grade 8 Standard*</p>
<p>4.G.8 Identify and describe line symmetry in two-dimensional shapes.</p>	<p>4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>
<p>4.G.9 Predict and validate the results of partitioning, folding, and combining two- and three-dimensional shapes.</p>	<p>Omitted</p>

Measurement	
<p>4.M.1 Demonstrate an understanding of such attributes as length, area, weight, and volume, and select the appropriate type of unit for measuring each attribute. :</p> <p>4.M.2 Carry out simple unit conversions within a system of measurement, e.g., hours to minutes, cents to dollars, yards to feet or inches, etc.</p> <p>4.M.3 Identify time to the minute on analog and digital clocks using a.m. and p.m. Compute elapsed time using a clock (e.g., hours and minutes since...) and using a calendar (e.g., days since...).</p> <p>4.M.4 Estimate and find area and perimeter of a rectangle, triangle, or irregular shape using diagrams, models, and grids or by measuring.</p> <p>4.M.5 Identify and use appropriate metric and English units and tools (e.g., ruler, angle ruler, graduated cylinder, thermometer) to estimate, measure, and solve problems involving length, area, volume, weight, time, angle size, and temperature.</p>	<p>4.MD Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p> <p>1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p> <div data-bbox="824 600 1451 743" style="border: 1px dashed gray; padding: 5px;"> <p>DESE Comment * Implies the use of US Customary units through the example.</p> </div> <p>2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p> <p>3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i></p>

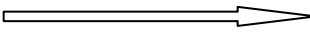
Data Analysis, Statistics & Probability	
4.D.1 Collect and organize data using observations, measurements, surveys, or experiments, and identify appropriate ways to display the data.	<p style="text-align: center;">*Moved to Common Core Grade 3 Standard*</p> <p>3.MD Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.(Excludes multiplicative comparison problems (problems involving notions of "times as much"))</p> <p>3.MD Represent and interpret data. 3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i></p> <p>4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</p>
4.D.2 Match a representation of a data set with the actual set of data.	Omitted
4.D.3 Construct, draw conclusions, and make predictions from various representations of data sets, including tables, bar graphs, circle graphs, pictographs, line graphs, line plots, and tallies.	<p>4.MD Represent and interpret data. 4. Make a line plot to display a data set of measurements in fractions of a unit ($1/2$, $1/4$, $1/8$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p> <div style="border: 1px dashed gray; padding: 10px; margin-top: 10px;"> <p>DESE Comment * Requires solving data problems using addition and subtraction of data represented fractionally.</p> </div>
4.D.4 Represent the possible outcomes for a simple probability situation, e.g., the probability of drawing a red marble from a bag containing three red marbles and four green marbles.	*Moved to Common Core Grade 7 Standard*
4.D.5 List and count the number of possible combinations of objects from three sets, e.g., how many different outfits can one make from a set of three shirts, a set of two skirts, and a set of two hats?	*Moved to Common Core Grade 7 Standard*
4.D.6 Classify outcomes as certain, likely, unlikely, or impossible by designing and conducting experiments using concrete objects such as counters, number cubes, spinners, or coins.	*Moved to Common Core Grade 7 Standard*

Grade 5: Critical Areas

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

- (1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
- (2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
- (3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

The Standards for Mathematical Practice complement the content standards at each grade level so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise.

<p style="text-align: center;">Grade 5 Massachusetts Mathematics Standards</p>	<p style="text-align: center;">Grade 5 Common Core Standards</p>
<p>Number Sense & Operations</p>	
<p>5.N.1 Demonstrate an understanding of positive integer exponents, in particular, when used in powers of ten, e.g., 10 to the 2nd power, 10 to the 5th power.</p>	<p>5.NBT Understand the place value system.</p> <p>2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p>
<p>5.N.2 Demonstrate an understanding of place value through millions and thousandths.</p> <p>5.N.3 Represent and compare very large (millions) and very small (thousandths) positive numbers in various forms such as expanded notation without exponents, e.g., $9724 = 9 \times 1000 + 7 \times 100 + 2 \times 10 + 4$.</p>	<p>5.NBT Understand the place value system.</p> <p>1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</p> <div style="border: 1px dashed gray; padding: 5px; margin: 5px 0;"> <p>DESE Comment * Defines place value in terms of digit placement to the right (10 times as much) or to left (1/10 as much).</p> </div> <p>2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p> <div style="border: 1px dashed gray; padding: 5px; margin: 5px 0;"> <p>DESE Comment * Requires explanation of patterns formed by multiplying by multiples of 10.</p> </div> <p>3. Read, write, and compare decimals to thousandths.</p> <p>A. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.</p> <div style="border: 1px dashed gray; padding: 5px; margin: 5px 0;"> <p>DESE Comment * Requires using symbols to compare decimals.</p> </div> <p>B. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p> <p>4. Use place value understanding to round decimals to any place.</p>
<p>Standard Change </p>	<p>5.NBT Understand the place value system</p> <p>4. Use place value understanding to round decimals to any place.</p>

<p>5.N.4 Demonstrate an understanding of fractions as a ratio of whole numbers, as parts of unit wholes, as parts of a collection, and as locations on the number line.</p>	<p>2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. <i>For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$</i></p>
<p>5.N.5 Identify and determine common equivalent fractions (with denominators 2, 4, 5, 10) and mixed numbers (with denominators 2, 4, 5, 10), decimals, and percents (through 100%), e.g. $3/4 = 0.75 = 75\%$.</p> <div style="border: 1px dashed black; padding: 5px; margin-top: 10px;"> <p>Comment * Percents move to grade 6</p> </div>	<p style="text-align: center;">*Moved to Common Core Grade 3 Standard*</p> <p>3. NF Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.</p> <p>1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p> <p>2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>A. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p> <p>B. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</p> <p>3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>A. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p>B. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>C. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. <i>Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.</i></p>
<p>5.N.6 Find and position whole numbers, positive fractions, positive mixed numbers, and positive decimals, on the number line.</p>	<p style="text-align: center;">*Moved to Common Core Grade 4 Standard*</p> <p>4.NF Understand decimal notation for fractions, and compare decimal fractions.</p> <p>5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.² <i>For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)</i></p> <p>6. Use decimal notation for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</i></p>

<p>5.N.7 Compare and order whole numbers, positive fractions, positive mixed numbers, positive decimals, and percents.</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>Comment * Percents move to grade 6</p> </div>	<p>5.NBT Understand the place value system.</p> <p>3. B. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p> <p>4. Use place value understanding to round decimals to any place.</p>
<p>5.N.8 Apply number theory concepts of common factor, common multiple, and divisibility rules for 2, 3, 4, 5, and 10 to the solution of problems. Demonstrate an understanding of prime and composite numbers:</p>	<p style="text-align: center;">*Moved to Common Core Grade 4 Standard*</p> <p>4.OA Gain familiarity with factors and multiples.</p> <p>4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p>
<p>5.N.9 Solve problems involving multiplication and division of whole numbers, and multiplication of positive fractions with whole numbers.</p>	<p>5.NF Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</p> <p>3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i></p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 20px;"> <p>DESE Comment * Interprets fractions divisions as division of numerators by denominators.</p> </div>

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

A. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

B. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by:

DESE Comment

* Scaling requires explanation of the results of multiplying by a fraction greater or less than 1.

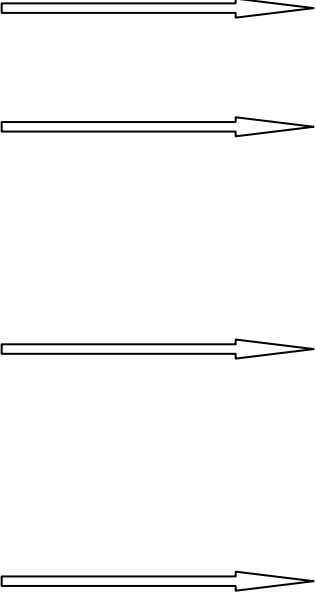
A. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

B. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

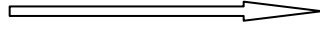
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

DESE Comment

* Requires involving fractions in real world problems.

<p>Standard Change</p> 	<p>7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.¹</p> <p>A. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. <i>For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.</i></p> <p>B. Interpret division of a whole number by a unit fraction, and compute such quotients. <i>For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.</i></p> <p>C. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$-cup servings are in 2 cups of raisins?</i></p> <p>¹ Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.</p>
<p>5.N.10 Demonstrate an understanding of how parentheses affect expressions involving addition, subtraction, and multiplication, and use that understanding to solve problems, e.g. $3 \times (4+2) = 3 \times 6$.</p>	<p>5.OA Write and interpret numerical expressions.</p> <p>1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</p> <p>2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.</i></p>
<p>5.N.11 Demonstrate an understanding of the inverse relationship of addition and subtraction, and use that understanding to simplify computation and solve problems.</p>	<p>Omitted</p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>Comment * Skills may be needed to solve problems.</p> </div>

Standard Change



5.N.12 Accurately and efficiently add, and subtract whole numbers and positive decimals. Multiply and divide (using double digit divisors) whole numbers. Multiply positive decimals with whole numbers.

5.NBT Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

DESE Comment

* Requires using strategies based on place value and modeling of division calculations with whole numbers.

7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

DESE Comment

* Stresses the importance of using concrete models, drawings, and strategies based on place value to perform operations with decimals and requires explanation of reasoning.

5.N.13 Accurately and efficiently add and subtract positive fractions and mixed numbers with like denominators and with unlike denominators (2, 4, 5, 10 only); multiply positive fractions with whole numbers. Simplify fractions in cases when both the numerator and the denominator have 2, 3, 4, 5 or 10 as a common factor.. Simplify fractions.

5.N.14 Estimate results of computations with whole numbers, positive fractions, and positive decimals. Estimate products of whole numbers and products of positive decimals with whole numbers. Use a variety of strategies and judge the reasonableness of the answer.

5.NF Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*

DESE Comment

* Requires addition and subtraction of fractions of any denominator.

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.*

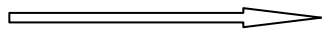
DESE Comment

* Specifies use of benchmark fractions for mental estimates and solving word problems involving fractions.

Aligns with Common Core Standard 5.NF about fractions.

The Number System

Standard Change



5.NS Gain Familiarity with concepts of positive and negative integers

MA.1 Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit.

DESE Comment

* Massachusetts addition. Introduces the concept of negatives to complement introduction of the coordinate plane.

Patterns, Relations & Algebra	
5.P.1 Analyze and determine the rules for extending symbolic, arithmetic, and geometric patterns and progressions, e.g., ABCC; 1, 5, 9, 13 ...; 3, 9, 27,	<p align="center">*Moved to Common Core Grade 4 Standard*</p> <p>4.OA Generate and analyze patterns.</p> <p>5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i></p>
5.P.2 Replace variables with given values and evaluate/simplify, e.g., $2(m) + 3$ when $m = 4$.	Omitted
5.P.3 Use the properties of equality to solve problems with whole numbers, e.g., if $c + 7 = 13$, then $c = 13 - 7$, therefore $c = 6$; if $3 \times c = 15$, then $3 \times c$ divided by $3 = 15$ divided by 3 , therefore $c = 5$.	Omitted
5.P.4 Represent real situations and mathematical relationships with concrete models, tables, graphs, and rules in words and with symbols, e.g., input-output tables.	<p>5.OA Analyze patterns and relationships.</p> <p>3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. <i>For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</i></p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment</p> <p>* Requires generation of numerical patterns using two rules to form and graph ordered pairs, and requires informal explanation.</p> </div>
5.P.5 Solve problems involving proportional relationships using concrete models, tables, graphs, and paper and pencil methods.	Omitted
5.P.6 Interpret graphs that represent the relationship between two variables in everyday situations.	<p>5.G Graph points on the coordinate plane to solve real-world and mathematical problems.</p> <p>2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p>

Geometry	
5.G.1 Identify, describe and compare special types of triangles (isosceles, equilateral, right) and quadrilaterals (square, rectangle, parallelogram, rhombus, trapezoid) e.g. recognize that all equilateral triangles are isosceles but not all isosceles triangles are equilateral.	<p>5. G Classify two-dimensional figures into categories based on their properties.</p> <p>3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</p> <p>4. Classify two-dimensional figures in a hierarchy based on properties.</p>
5.G.2 Identify, describe and compare special types of three-dimensional shapes (e.g., cubes, prisms, spheres, and pyramids) based on their properties, such as edges and faces.	<p>*Moved to Common Core Grade 3 Standard*</p> <p>3.G Reason with shapes and their attributes.</p> <p>1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p>
5.G.3 Identify relationships among points, and lines. e.g., intersecting, parallel, perpendicular.	<p>*Moved to Common Core Grade 4 Standard*</p> <p>4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</p> <p>1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p>
5.G.4 Using ordered pairs of whole numbers (including zero) graph, locate and identify points and describe paths on the Cartesian coordinate plane.	<p>5.G Graph points on the coordinate plane to solve real-world and mathematical problems.</p> <p>1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</p> <p>2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p>
5.G.5 Describe, and perform transformations on two-dimensional shapes, e.g., translations, rotations, and reflections.	Omitted

<p>5.G.6 Identify and describe line symmetry in two dimensional shapes, including shapes that have multiple lines of symmetry.</p>	<p>*Moved to Common Core Grade 4 Standard* 4.G Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>
<p>5.G.7 Determine if two shapes or two quadrilaterals are congruent by measuring sides or a combination of sides and angles, as necessary; or by motions or series of motions, e.g., translations, rotations, and reflections.</p>	<p>Omitted</p>
<p>Measurement</p>	
<p>5.M.1 Apply the concepts of perimeter and area to the solution of problems with triangles and rectangles. Apply formulas where appropriate.</p>	<p>5.NF Apply and extend previous understandings of multiplication and division to multiply and divide fractions. B. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p>
<p>5.M.2 Identify, measure, describe, classify, and draw various angles. Draw triangles given two sides and the angle between them, or given two angles and the side between them. e.g. draw a triangle with one right angle and two sides congruent.</p>	<p>*Moved to Common Core Grade 4 Standard* 4.MD Geometric measurement: understand concepts of angle and measure angles. 5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: A. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a "one-degree angle," and can be used to measure angles. B. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. 6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measurement.</p>
<p>5.M.3 Solve problems involving simple unit conversions within a system of measurement.</p>	<p>5.MD Convert like measurement units within a given measurement system. 1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p>

5.M.4 Find volumes and surface areas of rectangular prisms.

Comment

* Surface area moved to grade 6.

5.MD Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

A. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

DESE Comment

* Specifies use of the unit cube.

B. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

DESE Comment

* Emphasizes the relationship of volume with multiplication and addition.

A. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

B. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

C. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

<p>5.M.5 Find the sum of the interior angles in triangles with and without measuring the angles.</p>	<p style="text-align: center;">*Common Core Grade 4 Standard*</p> <p>4.MD Geometric measurement: understand concepts of angle and measure angles.</p> <p>5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <p>A. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a "one-degree angle," and can be used to measure angles.</p> <p>B. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.</p> <p>6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p> <p>7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measurement.</p>
<p>Data Analysis, Statistics & Probability</p>	
<p>5.D.1 Given a set of data, find the median, mean, mode, maximum and minimum, and range. Apply these to solutions of problems.</p>	<p>Omitted</p>
<p>5.D.2 Construct and interpret line plots, line graphs, and bar graphs. Interpret and label circle graphs</p>	<p>5.MD Represent and interpret data.</p> <p>2. Make a line plot to display a data set of measurements in fractions of a unit ($1/2, 1/4, 1/8$). Use operations on fractions for this grade to solve problems involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i></p> <div style="border: 1px dashed gray; padding: 5px; margin-top: 10px;"> <p>DESE Comment * Requires fractional representation of data.</p> </div>
<p>5.D.4 Predict the probability of outcomes of simple experiments (e.g., tossing a coin, rolling a die, also known as a number cube) and test the predictions.</p>	<p>Omitted</p>